Digital Signal Processing

http://kom.aau.dk/~zt/cources/Digital_signal_processing/

Exercises of Lecture 5 (MM5)

Exercise 5.1.

Compute the DFT of each of the following finite-length sequences considered to be o length N (where N is even).

(a) $x[n] = \delta[n]$

(b) $x[n] = \delta[n - n_0], \quad 0 \le n_0 \le N - 1$

(c) $x[n] = \begin{cases} a^n, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$

Exercise 5.2.

Suppose we have two 4-point sequences $x[n]$ and $h[n]$ as follows:

$$x[n] = \cos\left(\frac{\pi n}{2}\right), \qquad n = 0, 1, 2, 3,$$

$$h[n] = 2^n, \qquad n = 0, 1, 2, 3.$$

(a) Calculate the 4-point DFT $X[k]$.

(b) Calculate the 4-point DFT $H[k]$.

(c) Calculate $y[n] = x[n] \circledast_4 h[n]$ by doing the circular convolution directly.

(d) Calculate $y[n]$ of part (c) by multiplying the DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.

Exercise 5.3.

A signal $x[n]$ has been filtered by a linear time-invariant system with impulse response $h[n] = \delta[n] - \frac{1}{2}\delta[n-1]$. The output of the system is the distorted signal $y[n]$. Theoretically, $x[n]$ can be recovered from $y[n]$ through an inverse filter having a system function equal to the reciprocal of the system function of the distorting filter.

(a) Determine the z-transform $H(z)$ and the four-point DFT $H[k]$ of the impulse response $h[n]$.

(b) Let $H_i(z)$ denote the system function of the inverse filter and let $h_i[n]$ be the corresponding impulse response. Determine $h_i[n]$. Is this an FIR or an IIR filter?

(c) If $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$, calculate $y[n] = x[n] * h[n]$ by doing the linear convolution directly.

(d) Calculate $y[n]$ of Question (c) by multiplying the four-point DFTs of $x[n]$ and $h[n]$ and performing an inverse DFT.

(e) Determine the z-transform Y(z) and determine the inverse z-transform of $Y(z) \cdot H_i(z)$.