Digital Signal Processing

http://kom.aau.dk/~zt/cources/Digital_signal_processing/

Solutions of Lecture 4 (MM4)

Exercise 4.1. Design a discrete-time filter that meets specifications

$$0.87 \le |H(e^{j\omega})| \le 1, \qquad 0 \le |\omega| \le 0.25\pi,$$
$$|H(e^{j\omega})| \le 0.21, \qquad 0.35 \le |\omega| \le \pi$$

by using the impulse invariance method to a continuous-time Butterworth filter having magnitude-squared function

$$|H_c(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$$

Determine the integer order N and the quantity Ω_c such that the continuous-time Butterworth filter exactly meets the specifications. (Assume that *Td*=1)

Solution:

$$1+(\frac{0.25\pi}{\Omega_c})^{2N} = (\frac{1}{0.87})^2$$
$$1+(\frac{0.35\pi}{\Omega_c})^{2N} = (\frac{1}{0.21})^2$$
$$2N = \frac{\log \frac{(\frac{1}{0.87})^2 - 1}{(\frac{1}{0.21})^2 - 1}}{\log \frac{0.25}{0.35}}$$
$$N = 6.259$$
$$\Omega_c = \frac{0.25\pi}{((\frac{1}{0.87})^2 - 1)^{1/2N}} = 0.86$$

So

$$N = 7$$
$$\Omega_c = \frac{0.25\pi}{((\frac{1}{0.87})^2 - 1)^{1/2N}} = 0.8518$$

Exercise 4.2. We wish to use the Kaiser window method to design a discrete-time filter with generalized linear phase that meets specifications of the following form :

$$|H(e^{j\omega})| \le 0.01, \qquad 0 \le |\omega| \le 0.25\pi$$
$$0.95 \le |H(e^{j\omega})| \le 1.05, \qquad 0.35\pi \le |\omega| \le 0.6\pi$$
$$|H(e^{j\omega})| \le 0.01, \qquad 0.65\pi \le |\omega| \le \pi$$

(a) Determine the minimum length (M+1) of the impulse response and the value of the Kaiser window parameter β for a filter that meets the preceding specifications.

(b) What is the delay of the filter?

(c) Determine the ideal impulse response $h_d[n]$ to which the Kaiser window should be applied.

Solution:

(a) M+1=91, $\beta = 3.395$

(b) M/2=45

(c)

$$h_d[n] = \frac{\sin[0.625\pi(n-45)]}{\pi(n-45)} - \frac{\sin[0.3\pi(n-45)]}{\pi(n-45)}$$

Exercise 4.3. We wish to design an FIR lowpass filter satisfying the specifications

$$0.95 < H(e^{j\omega}) < 1.05, \qquad 0 \le |\omega| \le 0.25\pi$$

$$-0.1 < H(e^{j\omega}) < 0.1, \qquad 0.35\pi \le |\omega| \le \pi$$

by applying a window w[n] to the impulse response $h_d[n]$ for the ideal discrete-time lowpass filter with cutoff $\omega_c = 0.3\pi$. Which of the windows listed in Table 7.1 can be used to meet this specification?

Solution:

The Hamming, Hanning, and Blackman windows may be used, since

$$20\log_{10}\delta = 20\log_{10} 0.05 = -26$$