# Digital Signal Processing, Fall 2009

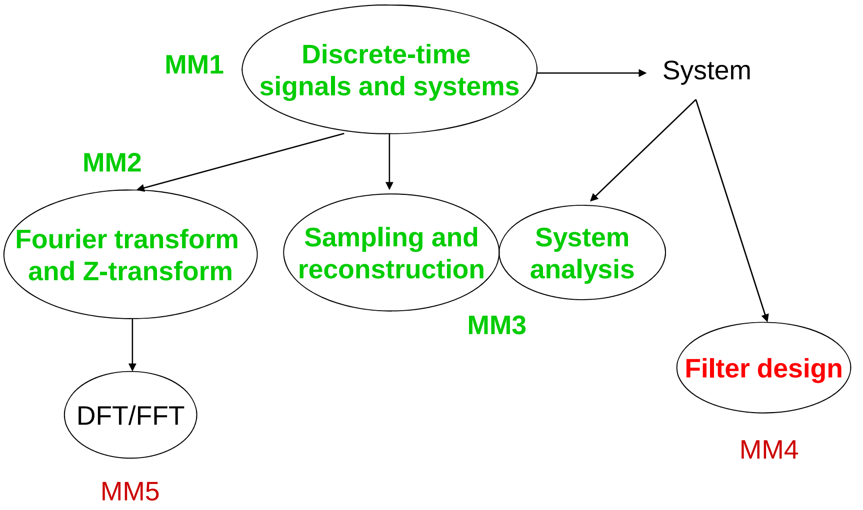
## Lecture 4: Filter Design


Zheng-Hua Tan

Department of Electronic Systems
Aalborg University, Denmark
zt@es.aau.dk




## Course at a glance



# Part I: Filter design

- Filter design
- IIR filter design
- FIR filter design

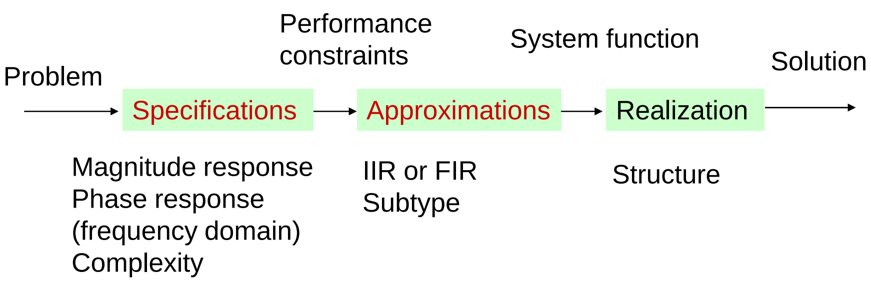


AALBORG UNIVERSITY

# Filter design process

Filter, in broader sense, covers any system.

Three design steps





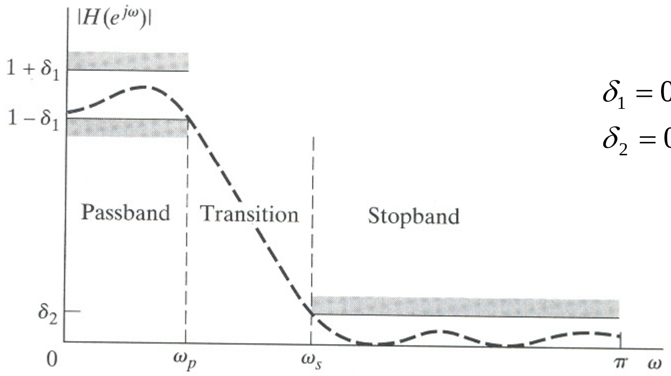

AALBORG UNIVERSITY

# Specifications – an example

- Specifications for a discrete-time lowpass filter

$$1-0.01 \le |H(e^{j\omega})| \le 1+0.01, \quad 0 \le \omega \le \omega_p$$

$$|H(e^{j\omega})| \le 0.001, \quad \omega \ge \omega_s$$




$$\delta_1 = 0.01$$

$$\delta_2 = 0.001$$

# Specifications of frequency response

- Typical lowpass filter specifications in terms of tolerable
  - Passband distortion, as smallest as possible
  - Stopband attenuation, as greatest as possible
  - Width of transition band: as narrowest as possible

- Improving one often worsens others → a tradeoff
- Increasing filter order improves all



AALBORG UNIVERSITY

# DT filter for CT signals

- DT filter for the processing of CT signals
  - Bandlimited input signal
  - High enough sampling frequency
- Then, specifications (often given in frequency domain) conversion is straightforward

$$H_{eff}(j\Omega) = \begin{cases} H(e^{j\Omega T}), & |\Omega| < \pi / T \\ 0, & |\Omega| > \pi / T \end{cases}$$

Signal is band-limited; $T$ is small enough.

$$H(e^{j\omega}) = H_{eff}(j\frac{\omega}{T}), \qquad |\omega| < \pi \qquad\qquad \omega = \Omega T$$

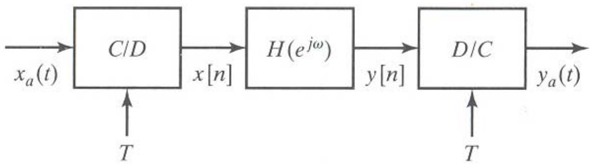


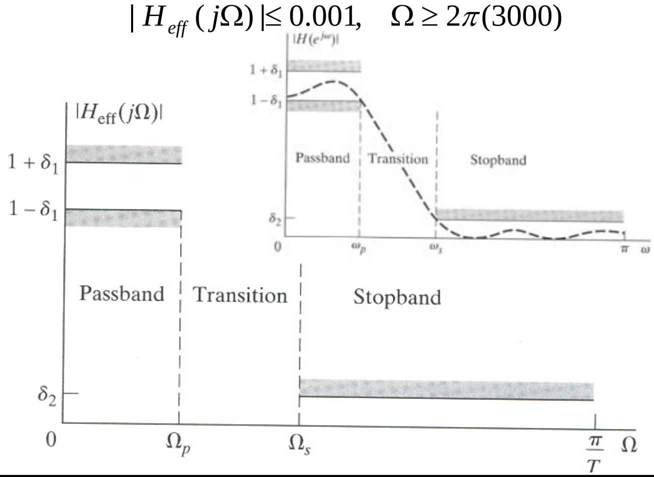

**Figure 7.1** Basic system for discrete-time filtering of continuous-time signals.

# Specifications – an example

- Specifications for a CT lowpass filter

$$1 - 0.01 \le |H_{eff}(j\Omega)| \le 1 + 0.01, \quad 0 \le \Omega \le 2\pi(2000)$$

$$|H_{eff}(j\Omega)| \le 0.001, \quad \Omega \ge 2\pi(3000)$$




$\omega = \Omega T$

$\delta_1 = 0.01$

$\delta_2 = 0.001$

$\Omega_p = 2\pi(2000)$

$\Omega_s = 2\pi(3000)$

## Design a filter

- Design goal: find system function to make frequency response meet the specifications (tolerances)
- Infinite impulse response filter
  - Poles insider unit circle due to causality and stability
  - Rational function approximation
- Finite impulse response filter
  - Linear phase is often required
  - Polynomial approximation



## E.g. IIR filter design

- For rational system function

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

find the system coefficients such that the corresponding frequency response

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}}$$

provides a good approximation to a desired response

$$H(e^{j\omega}) \approx H_{desired}(e^{j\omega}) \longrightarrow$$

H(z)
•Rational system function
•Stable
•causal

## FIR and IIR

IIR

- Rational system function
- Poles + zeros
- Stable/unstable
- Hard to control phase
- Low order (4-20)
- Designed on the basis of analog filter

FIR

- Polynomial system function
- Zeros
- Stable
- Easy to get linear phase
- High order (20-2000)
- Unrelated to analog filter



## FIR or IIR

- Whether FIR or IIR often depends on the phase requirements
- Design principle
  - If GLP is essential → FIR
  - If not → IIR preferable (can meet specifications with lower complexity)

## Part II: IIR Filter design

- Filter design
- IIR filter design
  - Analog filter design
  - IIR filter design by impulse invariance
- FIR filter design





## Design IIR filter based on analog filter

- The mapping is direct

$$H_{eff}(j\Omega) = \begin{cases} H(e^{j\Omega T}), & |\Omega| < \pi / T \\ 0, & |\Omega| > \pi / T \end{cases}$$

Signal is band-limited; *T* is small enough.

$$H(e^{j\omega}) = H_{eff}(j\frac{\omega}{T}), \qquad |\omega| < \pi$$

- Advanced analog filter design techniques

→ Designing DT filter by transforming prototype CT filter:
  - Transform (map) DT specifications to analog
  - Design analog filter
  - Inverse-transform analog filter to DT

## Transformation method

- Transform (map) DT specifications to analog

  $\omega = \Omega T$

- Design analog filter

  $H_c(s)$ or $h_c(t)$

- Inverse-transform to DT

  $H(z)$ or $h[n]$

$s = \sigma + j\Omega$

$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$

$H(j\Omega) = \int_{-\infty}^{\infty} h(t) e^{-j\Omega t} dt$

$z = re^{j\omega}$

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

• The imaginary axis of the s-plane → the unit circle of the z-plane
• Poles in the left half of the s-plane → poles inside the unit circle in the z-plane (stable)



## Part II-A: Analog Filter design

- Filter design
- IIR filter design
  - Analog filter design
  - IIR filter design by impulse invariance
- FIR filter design

# Analog filter design

- Butterworth
- Chebyshev I
- Chebyshev II
- Ellipical



# Butterworth lowpass filters

- The magnitude response
  - Maximally flat in the passband
  - Monotonic in both passband and stopband
- The squared magnitude response

$$|H_c(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$$

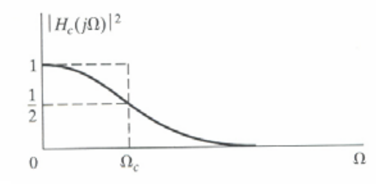



**Figure B.1** Magnitude-squared function for continuous-time Butterworth filter.

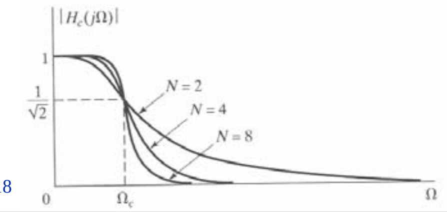



**Figure B.2** Dependence of Butterworth magnitude characteristics on the order $N$.

# Poles in s-plane

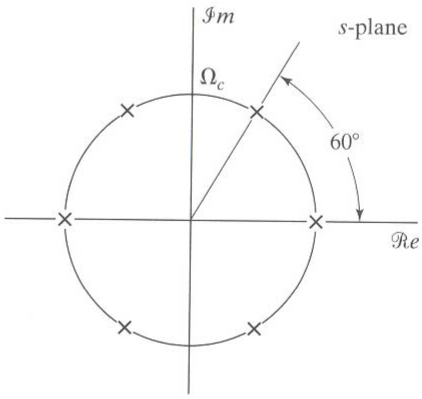



**Figure B.3** *s*-plane pole locations for a third-order Butterworth filter



# Part II-B: Design by impulse invariance

- Filter design
- IIR filter design
  - Analog filter design
  - IIR filter design by impulse invariance
- FIR filter design

## Filter design by impulse invariance

- Impulse invariance: a method for obtaining a DT system whose $H(e^{j\omega})$ is determined by the $H_c(j\Omega)$ of a CT system.

$$h[n] = T_d h_c(nT_d)$$

$T_d$ - 'design' sampling interval

  - In DT filter design, the specifications are provided in the DT, so $T_d$ has no role. $T_d$ is included for discussion though. $T_d$ also has nothing to do with C/D and D/C conversion in Fig. 7.1, i.e. $T_d$ need not be the same as the sampling period T of the C/D and D/C conversion.



## Relationship btw frequency responses

- Impulse response

$$h[n] = T_d h_c(nT_d)$$

- Frequency response

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c(j\frac{\omega}{T_d} + j\frac{2\pi}{T_d}k)$$

sampling:

$$x[n] = x_c(nT)$$

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j\frac{\omega}{T} - j\frac{2\pi k}{T})$$

if the CT filter is bandlimited

$$H_c(j\Omega) = 0, \quad |\Omega| \geq \pi / T_d$$

then

$$H(e^{j\omega}) = H_c(j\frac{\omega}{T_d}), \quad |\omega| \leq \pi$$

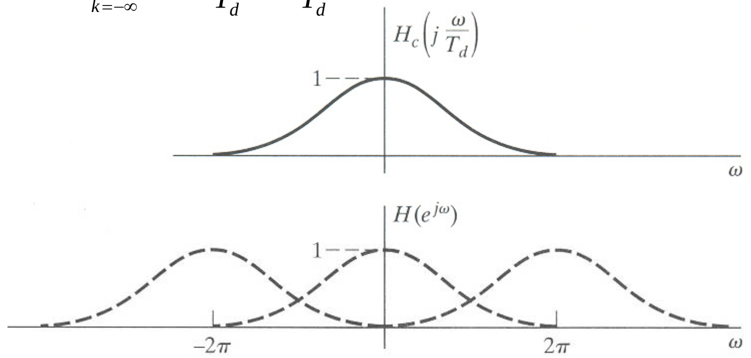
$$H(e^{j\omega}) = H_{eff}(j\frac{\omega}{T}), \ |\omega| < \pi$$

This is also the way to get CT filter specifications from $H(e^{j\omega})$ by applying the relation $\Omega = \omega / T_d$

# Aliasing in the impulse invariance design

$$h[n] = T_d h_c(nT_d)$$

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c\left(j\frac{\omega}{T_d} + j\frac{2\pi}{T_d}k\right)$$




The continuous-time filter may be designed to exceed the specifications, particularly in the stopband.



# Impulse invariance with a Butterworth filter

- Specifications

$$0.89125 \le |H(e^{j\omega})| \le 1, \quad 0 \le |\omega| \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.17783, \quad 0.3\pi \le |\omega| \le \pi$$

- Since the sampling interval $T_d$ cancels in the impulse invariance procedure, we choose $T_d$=1, so $\omega = \Omega$
- Magnitude function for a CT Butterworth filter

$$0.89125 \le |H_c(j\Omega)| \le 1, \quad 0 \le |\Omega| \le 0.2\pi$$

$$|H_c(j\Omega)| \le 0.17783, \quad 0.3\pi \le |\Omega| \le \pi$$

$$H_{eff}(j\Omega) = \begin{cases} H(e^{j\Omega T}), & |\Omega| < \pi/T \\ 0, & |\Omega| > \pi/T \end{cases}$$

$$H(e^{j\omega}) = H_{eff}\left(j\frac{\omega}{T}\right), \quad |\omega| < \pi$$

- Due to the monotonic function of Butterworth filter, we have

$$|H_c(j0.2\pi)| \ge 0.89125$$

$$|H_c(j0.3\pi)| \le 0.17783$$

## Impulse invariance with a Butterworth filter

- Squared magnitude function of a Butterworth filter

(2) $$|H_c(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$$ ← (1) $$|H_c(j0.2\pi)| \geq 0.89125$$ $$|H_c(j0.3\pi)| \leq 0.17783$$

(3)
$$1+\left(\frac{0.2\pi}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.89125}\right)^2$$
$$1+\left(\frac{0.3\pi}{\Omega_c}\right)^{2N} = \left(\frac{1}{0.17783}\right)^2$$

(5) $N = 6$

(6) $\Omega_c = 0.7032$

(4) $N = 5.8858$, $\Omega_c = 0.70474$

$$H_c(s)H_c(-s) = \frac{1}{1+(s/j\Omega_c)^{2N}}$$
$$= \frac{1}{1+(s/j0.7032)^{12}}$$



## Impulse invariance with a Butterworth filter

- 12 poles for the squared magnitude function
- The system function has the three pole pairs in the left half of the s-plane

Pole pair 1: $-0.182 \pm j(0.679)$,

Pole pair 2: $-0.497 \pm j(0.497)$,

Pole pair 3: $-0.679 \pm j(0.182)$.

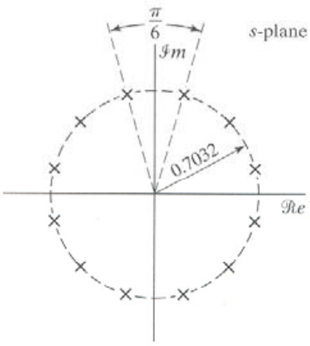



**Figure 7.4** s-plane locations for poles of $H_c(s)H_c(-s)$ for sixth-order Butterworth filter in Example 7.2.

## Impulse invariance with a Butterworth filter

To construct $H_c(s)$ from the magnitude-squared function $Hc(s)Hc(-s)$ , we choose the poles on the left-half-plane part of the s-plane to obtain a stable and causal filter.

$$H_c(s) = \frac{0.12093}{(s^2 + 0.3640s + 0.4945)(s^2 + 0.9945s + 0.4945)(s^2 + 1.3585s + 0.4945)}$$

Express $H_c(s)$ as a partial fraction expansion, perform the transformation of Eq. (7.12):

$$H(z) = \frac{0.2871 - 0.4466z^{-1}}{(1 - 1.2971z^{-1} + 0.6949z^{-2})}$$
$$+ \frac{-2.1428 + 1.1455z^{-1}}{(1 - 1.0691z^{-1} + 0.3699z^{-2})} + \frac{1.8557 - 0.6303z^{-1}}{(1 - 0.9972z^{-1} + 0.2570z^{-2})}$$



## Impulse invariance with a Butterworth filter

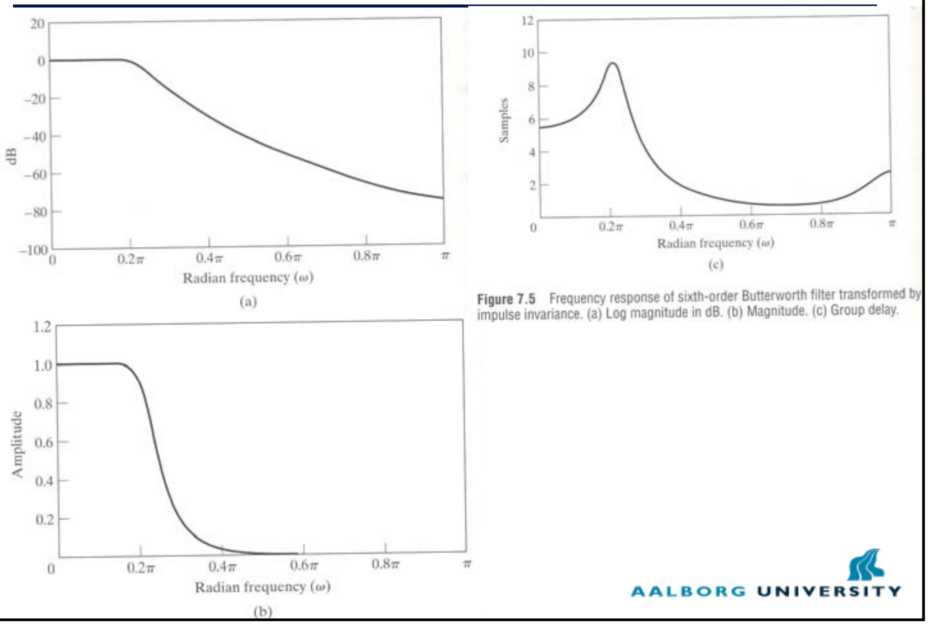



Figure 7.5 Frequency response of sixth-order Butterworth filter transformed by impulse invariance. (a) Log magnitude in dB. (b) Magnitude. (c) Group delay.

## Part III: FIR filter design

- Filter design
- IIR filter design
- FIR filter design
  - Commonly used windows
  - Generalized linear-phase FIR filter
  - The Kaiser window filter design method



## FIR filter design

- Design problem: the FIR system function

$$H(z) = \sum_{k=0}^{M} b_k z^{-k}$$

$$h[n] = \begin{cases} b_n, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

- Start from impulse response directly

$$H(z) = h[0] + h[1]z^{-1} + \dots + h[M]z^{-M}$$

Find

- the degree $M$ and
- the filter coefficients $h[k]$

to approximate a desired frequency response

## Lowpass filter – as an example

- Ideal lowpass filter

$$H_{lp}(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c \\ 0, & \omega_c < \omega \le \pi \end{cases} \qquad h[n] = \frac{\sin \omega_c n}{\pi n}, \quad -\infty < n < \infty$$

- IIR filter: based on transformations of CT IIR system into DT ones.

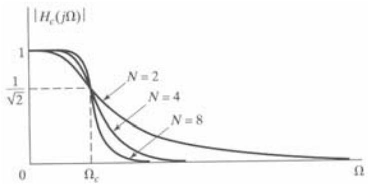



Figure B.2 Dependence of Butterworth magnitude characteristics on the order N.

$$|H_c(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$$

poles

- FIR filter: how? $h[n]$ is non-causal, infinite!



## Design by windowing

- Desired frequency responses are often piecewise-constant with discontinuities at the boundaries between bands, resulting in non-causal and infinte impulse response extending from $-\infty$ to $\infty$, but

$$n \to \pm\infty, \qquad h_d[n] \to 0$$

- So, the most straightforward method is to truncate the ideal response by windowing and do time-shifting:

$$g[n] = \begin{cases} h_d[n], & |n| \le M \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = g[n-M]$$

# Design by windowing

After Champagne & Labeau, DSP Class Notes 2002.

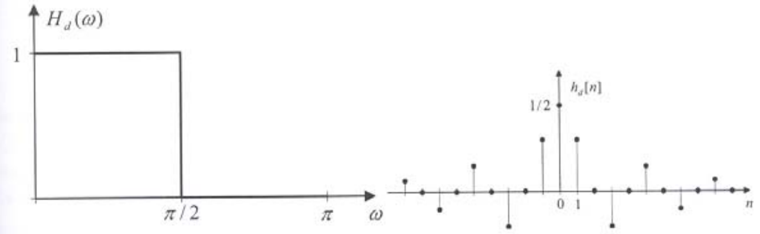



Figure 9.17: Desired (a) frequency response and (b) impulse response for example 9.7

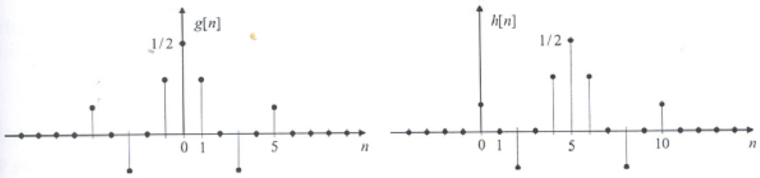



Figure 9.18: (a) Windowed impulse response $g[n]$ and (b) shifted windowed impulse response $h[n]$ for example 9.7

# Design by rectangular window

- In general,

$$h[n] = h_d[n]w[n]$$

For simple truncation, the window is the rectangular window

$$w[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

$$H(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\theta})H(e^{j(\omega-\theta)})d\theta$$

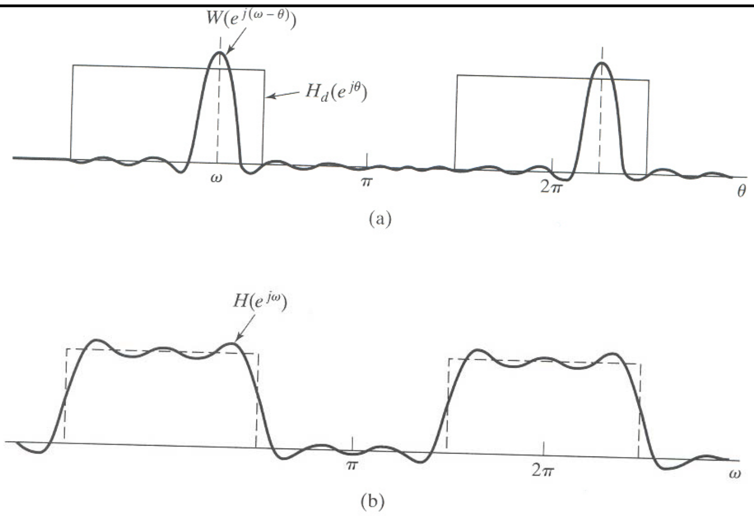



**Figure 7.19** (a) Convolution process implied by truncation of the ideal impulse response. (b) Typical approximation resulting from windowing the ideal impulse response.

$w[n]=1, \quad -\infty < n < \infty$, then what? $\quad W(e^{j\omega}) = \sum_{r=-\infty}^{\infty} 2\pi\delta(\omega+2\pi r)$

$W(e^{j\omega})$ should be narrow band

# Requirements on the window

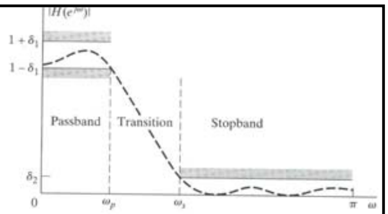
$w[n]=1, \quad -\infty < n < \infty, \qquad W(e^{j\omega}) = \sum_{r=-\infty}^{\infty} 2\pi\delta(\omega+2\pi r)$

$W(e^{j\omega})$ should be narrow band

- Requirements
  - $W(e^{j\omega})$ approximates an impulse to faithfully reproduce the desired frequency response
  - $w[n]$ as short as possible in duration (the order of the filter) to minimize computation in the implementation of the filter
  - → Conflicting

Take the rectangular window as an example.

## Rectangular window



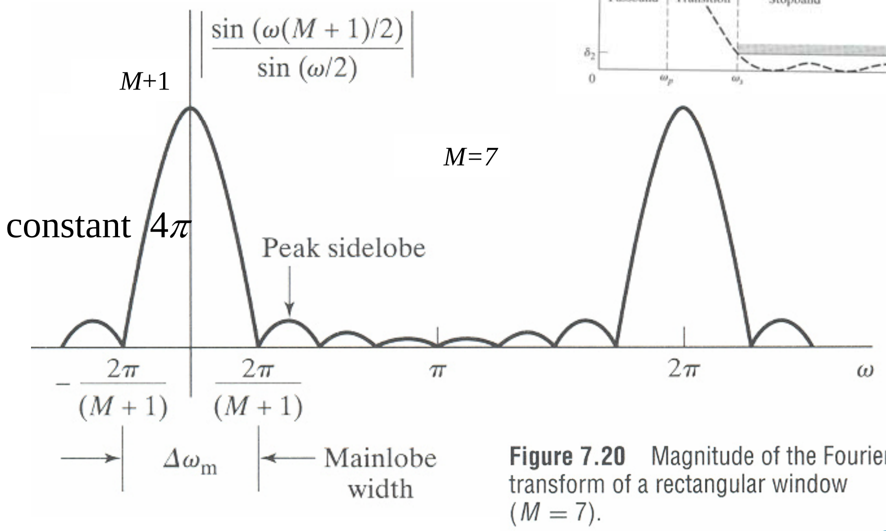




**Figure 7.20** Magnitude of the Fourier transform of a rectangular window ($M = 7$).



## Part III-A: Commonly used windows

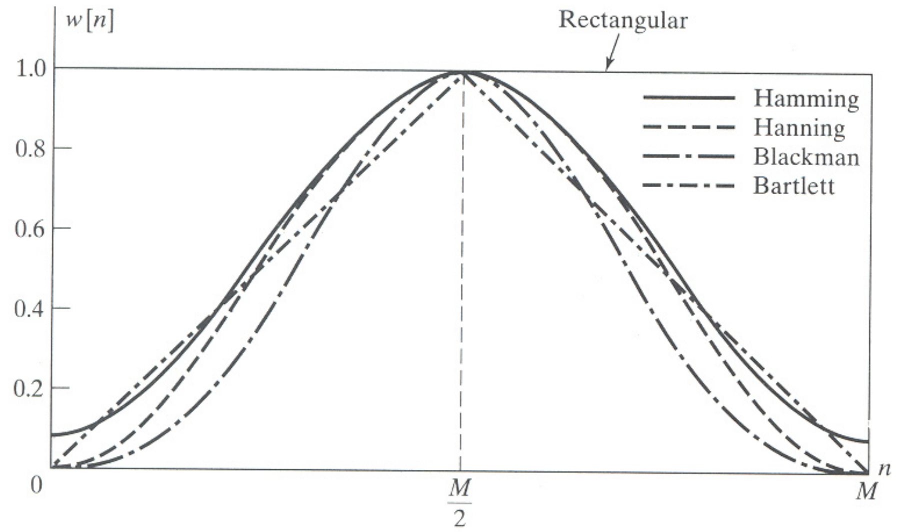
- Filter design
- IIR filter design
- FIR filter design
  - Commonly used windows
  - Generalized linear-phase FIR filter
  - The Kaiser window filter design method

## Standard windows – time domain

- Rectangular $w[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$
- Triangular $w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2 \\ 2 - 2n/M, & M/2 \le n \le M \\ 0, & \text{otherwise} \end{cases}$
- Hanning $w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$
- Hamming $w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$
- Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

## Standard windows – figure




Plotted for convenience. In fact, the window is defined only at integer values of *n*.

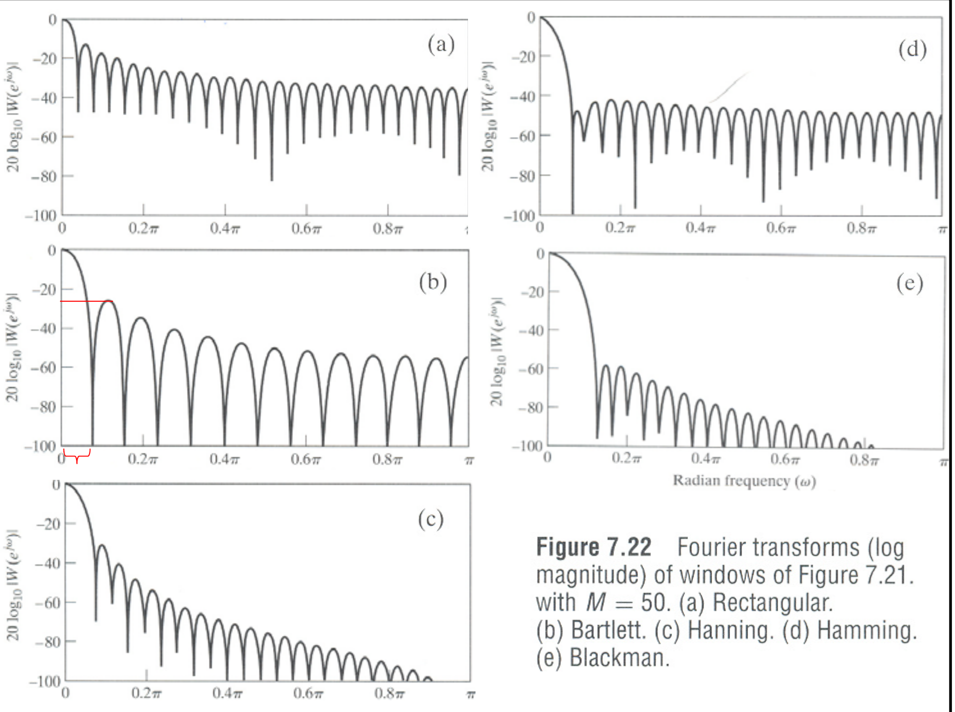



**Figure 7.22** Fourier transforms (log magnitude) of windows of Figure 7.21. with $M = 50$. (a) Rectangular. (b) Bartlett. (c) Hanning. (d) Hamming. (e) Blackman.

# Standard windows – comparison

Magnitude of side lobes vs width of main lobe

**TABLE 7.1** COMPARISON OF COMMONLY USED WINDOWS

| Type of Window | Peak Side-Lobe Amplitude (Relative) | Approximate Width of Main Lobe | Peak Approximation Error, $20 \log_{10} \delta$ (dB) | Equivalent Kaiser Window, $\beta$ | Transition Width of Equivalent Kaiser Window |
|---|---|---|---|---|---|
| Rectangular | −13 | $4\pi/(M+1)$ | −21 | 0 | $1.81\pi/M$ |
| Bartlett | −25 | $8\pi/M$ | −25 | 1.33 | $2.37\pi/M$ |
| Hanning | −31 | $8\pi/M$ | −44 | 3.86 | $5.01\pi/M$ |
| Hamming | −41 | $8\pi/M$ | −53 | 4.86 | $6.27\pi/M$ |
| Blackman | −57 | $12\pi/M$ | −74 | 7.04 | $9.19\pi/M$ |

Independent of M!

# Part III-B: Linear-phase FIR filter

- Filter design
- IIR filter design
  - Analog filter design
  - IIR filter design by impulse invariance
- FIR filter design
  - Commonly used windows
  - Generalized linear-phase FIR filter
  - The Kaiser window filter design method



# Linear-phase FIR systems

- Generalized linear-phase system

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega\alpha + j\beta}$$

$A(e^{j\omega})$ is a real function of $\omega$,

$\alpha$ and $\beta$ are real constants

- *Causal FIR systems* have generalized linear-phase if h[n] satisfies the symmetry condition

$$h[M-n] = h[n], \quad n = 0,1,\ldots,M$$

or

$$h[M-n] = -h[n], \quad n = 0,1,\ldots,M$$

## Generalized linear phase FIR filter

- Often aim at designing causal systems with a generalized linear phase (stability is not a problem)
  - If the impulse response of the desired filter is symmetric about M/2, $h_d[M-n] = h_d[n]$
  - Choose windows being symmetric about the point M/2

$$w[n] = \begin{cases} w[M-n], & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

$$W(e^{j\omega}) = W_e(e^{j\omega})e^{-j\omega M/2}$$

$W_e(e^{j\omega})$ is a real, even function of *w*

→ the resulting frequency response will have a generalized linear phase

$$H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega M/2}$$

$A_e(e^{j\omega})$ is real and even



## Linear-phase lowpass filter – an example

Desired frequency response

$$H_{lp}(e^{j\omega}) = \begin{cases} e^{-j\omega M/2}, & |\omega| < \omega_c \\ 0, & \omega_c < \omega \le \pi \end{cases}$$

$$h_{lp}[n] = \frac{\sin[\omega_c(n-M/2)]}{\pi(n-M/2)}, \quad -\infty < n < \infty$$

so

$$h_{lp}[M-n] = h_{lp}[n]$$

apply a symmetric window → a linear-phase system

$$h[n] = h_{lp}[n]w[n] = \frac{\sin[\omega_c(n-M/2)]}{\pi(n-M/2)}w[n]$$

## Window method approximations

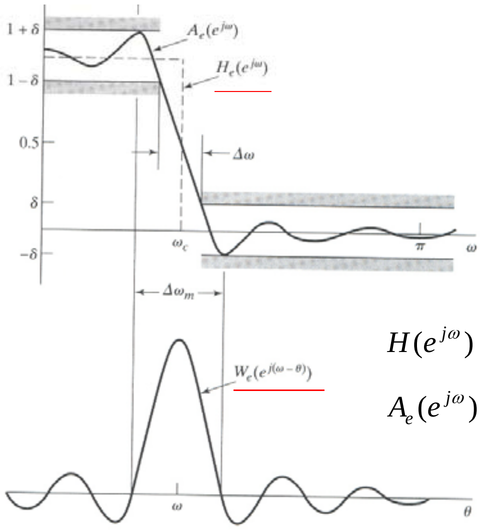



**Figure 7.23** Illustration of type of approximation obtained at a discontinuity of the ideal frequency response.

$$H_d(e^{j\omega}) = H_e(e^{j\omega})e^{-j\omega M/2}$$

$H_e(e^{j\omega})$ is real and even

$$W(e^{j\omega}) = W_e(e^{j\omega})e^{-j\omega M/2}$$

$W_e(e^{j\omega})$ is real and even

$$H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega M/2}$$

$$A_e(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_e(e^{j\theta})W_e(e^{j(\omega-\theta)})d\theta$$

AALBORG UNIVERSITY

## Key parameters

- To meet the requirement of FIR filter, choose
  - Shape of the window
  - Duration of the window
- Trail and error is not a satisfactory method to design filters → a simple formalization of the window method by Kaiser



AALBORG UNIVERSITY

# Part III-C: Kaiser window filter design

- Filter design
- IIR filter design
  - Analog filter design
  - IIR filter design by impulse invariance
- FIR filter design
  - Commonly used windows
  - Generalized linear-phase FIR filter
  - The Kaiser window filter design method



# The Kaiser window filter design method

- An easy way to find the trade-off between the main-lobe width and side-lobe area
- The Kaiser window

$$w[n] = \begin{cases} \dfrac{I_0[\beta(1-[(n-\alpha)/\alpha]^2)^{1/2}]}{I_0(\beta)}, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

$$\alpha = M/2$$

  - $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. $\beta \ge 0$ is an adjustable design parameter.
  - The length (M+1) and the shape parameter $\beta$ can be adjusted to trade side-lobe amplitude for main-lobe width (not possible for preceding windows!)

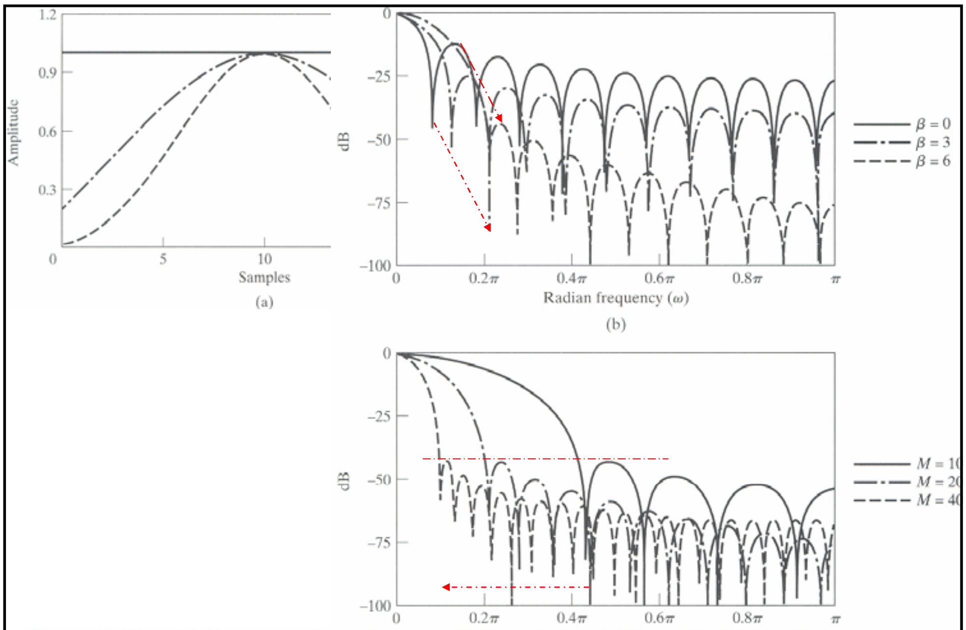



**Figure 7.24** (a) Kaiser windows for $\beta = 0$, 3, and 6 and $M = 20$. (b) Fourier transforms corresponding to windows in (a). (c) Fourier transforms of Kaiser windows with $\beta = 6$ and $M = 10$, 20, and 40.

# Design FIR filter by the Kaiser window

Calculate M and $\beta$ to meet the filter specification

- The peak approximation error $\delta$ is determined by $\beta$

Define $A = -20\log_{10}\delta$ then *(Peak error is fixed for other windows)*

$$\beta = \begin{cases} 0.1102(A-8.7), & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50 \\ 0.0, & A < 21 \end{cases}$$ *(Rectangular)*

- Passband cutoff frequency $\omega_p$ is determined by:

  $|H(e^{j\omega})| \ge 1-\delta$

  Stopband cutoff frequency $\omega_s$ by: $|H(e^{j\omega})| \le \delta$

  Transition width $\Delta\omega = \omega_s - \omega_p$

  $M$ must satisfy $M = \dfrac{A-8}{2.285\Delta\omega}$

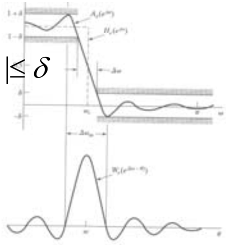


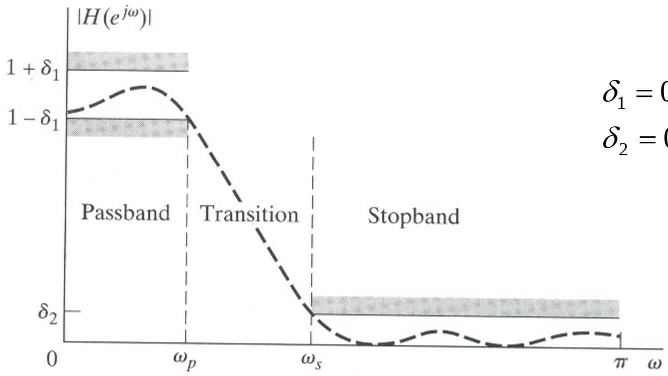
Figure 7.23 Illustration of type of approximation obtained at a discontinuity of the ideal frequency response

## A lowpass filter Specifications

- Specifications for a discrete-time lowpass filter

$$1-0.01 \le |H(e^{j\omega})| \le 1+0.01, \quad 0 \le \omega \le \omega_p = 0.4\pi$$

$$|H(e^{j\omega})| \le 0.001, \quad \omega \ge \omega_s = 0.6\pi$$




$$\delta_1 = 0.01$$
$$\delta_2 = 0.001$$

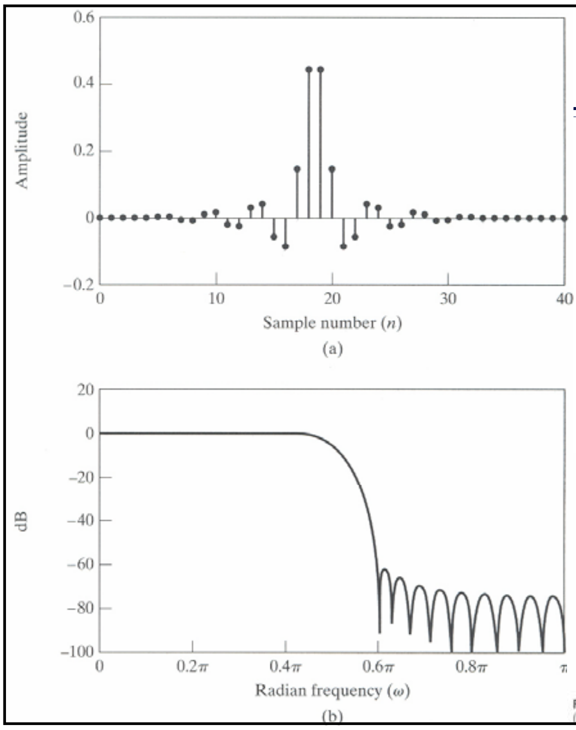
## Design the lowpass filter by Kaiser window

- Designing by window method indicating $\delta_1 = \delta_2$ , we must set $\delta = 0.001$
- Transition width $\Delta\omega = \omega_s - \omega_p = 0.2\pi$
- $A = -20\log_{10}\delta = 60$
- The two parameters: $\beta = 5.653, \quad M = 37$
- Cutoff frequency of the ideal lowpass filter

$$\omega_c = (\omega_s + \omega_p)/2 = 0.5\pi$$

- Impulse response

$$h[n] = \begin{cases} \dfrac{\sin\omega_c(n-\alpha)}{\pi(n-\alpha)} \cdot \dfrac{I_0[\beta(1-[(n-\alpha)/\alpha]^2)^{1/2}]}{I_0(\beta)}, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$




What is the group delay?

$M/2=18.5$

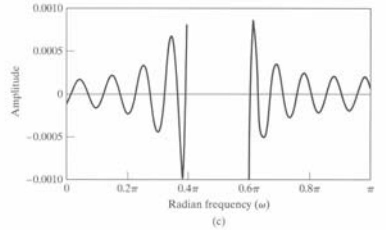



**Figure 7.25** Response functions for Example 7.8. (a) Impulse response ($M = 37$). (b) Log magnitude. (c) Approximation error.

# Summary

- Filter design
- IIR filter design
  - Analog filter design
  - IIR filter design by impulse invariance
- FIR filter design
  - Commonly used windows
  - Generalized linear-phase FIR filter
  - The Kaiser window filter design method

# Course at a glance