Digital Signal Processing

http://kom.aau.dk/~zt/cources/Digital_signal_processing/

Solutions 3 (MM3)

Exercise 3.1. If the sequence $x[n] = \cos(\frac{n\pi}{4}), -\infty < n < \infty$

is obtained by sampling the analog (continuous) signal $x_c(t) = \cos(\Omega_0 t), -\infty < t < \infty$ at a sampling rate of 1000 samples/s, what are two possible values of Ω_0 that could have resulted in the sequence $x[n]$?

Solution:

$$F_s = 1000 \qquad T = 1/1000$$
$$\omega = \Omega_0 T$$
$$\omega = \frac{\pi}{4} \rightarrow \Omega_0 = 250\pi$$

In general $\cos(\frac{\pi}{4}n) = \cos((2k\pi + \frac{\pi}{4})n)$

choose $\omega = 2\pi + \frac{\pi}{4} \rightarrow \Omega_0 = 2250\pi$

Exercise 3.2. The input signal, $x_c(t)$, given by $X_c(j\Omega) = 0, |\Omega| \geq 2\pi \times 10^4$

is subjected to a sampling system, using sampling period T, and having output $x[n]$:

$$x[n] = x_c(nT_s).$$

The signal $x[n]$ is then subjected to a digital filter, having impulse response $h[n]$, and generating the output $y[n] = T\sum_{k=-\infty}^{n} x[k]$.

a) What is the maximum allowable value of T if aliasing is to be avoided ? (so that x_c can be recovered from $x[n]$) ?

b) What is $h[n]$?

Solution:

a) $F_s > 2F_0 \rightarrow T_s < \frac{1}{2 \cdot 10^4}$

b)

$$y[n] = T_s \sum_{k=-\infty}^{n} x[k]$$

$$h[n] = T_s \sum_{k=-\infty}^{n} \delta[k] = T_s \cdot u[n]$$

Exercise 3.3 Consider the sampling and reconstruction system given below where $x(t)$ is given by the formula $x(t) = 10\cos(20\pi t - \pi/4) - 5\cos(50\pi t)$

a) What condition must be satisfied by the sampling rate to ensure $y(t) = x(t)$?

b) How should f_S be chosen, so that $y(t) = A + 10\cos(20\pi t - \pi/4)$?

c) What is the value of the constant A ?

Solution:

$a)$

$F_0 = 25Hz$

$\therefore \quad F_s > 50Hz \quad \text{to avoid aliasing}$

b)

In y(t), the 25 Hz component is aliased to 0 Hz. This is realised by choosing $F_s = 25Hz$ which at the same time assures that no aliasing for the 10 Hz component $10\cos(20\pi t - \pi/4)$.

c)

$$x[n] = 10\cos(20\pi n/25 - \pi/4) - 5\cos(50\pi n/25) = 10\cos(20\pi n/25 - \pi/4) - 5$$

Reconstruction of the signal:

$$y(t) = 10\cos(20\pi t - \pi/4) - 5$$

$$\therefore A = -5$$

Exercise 3.4. Consider a stable LTI system with input x[n] and output y[n]. The input and output satisfy the difference equation

$$y[n-1] - \frac{10}{3} y[n] + y[n+1] = x[n]$$

a) Find the system function
b) Plot the poles and zeros in the z-plane.

Solution:

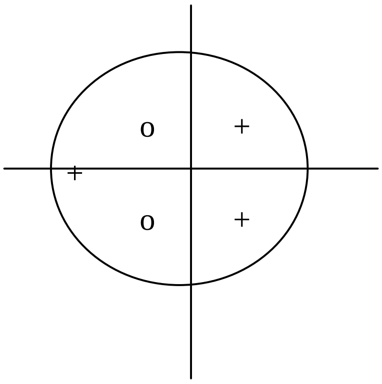
$a)$

$$H(z)=\frac{z^{-1}}{z^{-2}-\frac{10}{3}z^{-1}+1}=\frac{z^{-1}}{(z^{-1}-3)(z^{-1}-\frac{1}{3})}=\frac{z}{(1-3z)(1-\frac{1}{3}z)}$$

$b)$

$$\text{Poles}: z=3,\ \frac{1}{3};\quad \text{zeros}: z=0,\ \infty$$

Exercise 3.5. If the system function H(z) of a LTI system has a pole-zero diagram as shown in the following figure and the system is causal, can the inverse system Hi(z), where H(z)Hi(z)=1, be both causal and stable? Clearly justify your answer.



Solution:

$H_i(z)$ cannot be both causal and stable since the zero of H(z) at $z=\infty$ is a pole of $H_i(z)$ and the existence of a pole at $z=\infty$ implies that the system is not causal.