Digital Signal Processing

http://kom.aau.dk/~zt/cources/Digital_signal_processing/

Exercises of Lecture 3 (MM3)

Exercise 3.1. If the sequence $x[n] = \cos(\frac{n\pi}{4}), -\infty < n < \infty$

is obtained by sampling the analog (continuous) signal $x_c(t) = \cos(\Omega_0 t), -\infty < t < \infty$ at a sampling rate of 1000 samples/s, what are two possible values of Ω_0 that could have resulted in the sequence $x[n]$?

Exercise 3.2. The input signal, $x_c(t)$, given by $X_c(j\Omega) = 0, |\Omega| \geq 2\pi \times 10^4$

is subjected to a sampling system, using sampling period T, and having output $x[n]$:

$$x[n] = x_c(nT_s).$$

The signal $x[n]$ is then subjected to a digital filter, having impulse response $h[n]$, and generating the output $y[n] = T\sum_{k=-\infty}^{n} x[k]$.

a) What is the maximum allowable value of T if aliasing is to be avoided ? (so that x_c can be recovered from $x[n]$) ?

b) What is $h[n]$?

Exercise 3.3 Consider the sampling and reconstruction system given below where $x(t)$ is given by the formula $x(t) = 10\cos(20\pi t - \pi/4) - 5\cos(50\pi t)$

a) What condition must be satisfied by the sampling rate to ensure $y(t) = x(t)$?

b) How should f_s be chosen, so that $y(t) = A + 10\cos(20\pi t - \pi/4)$?

c) What is the value of the constant A ?

Exercise 3.4. Consider a stable LTI system with input x[n] and output y[n]. The input and output satisfy the difference equation

$$y[n-1]-\frac{10}{3}y[n]+y[n+1]=x[n]$$

a) Find the system function
b) Plot the poles and zeros in the z-plane.

Exercise 3.5. If the system function H(z) of a LTI system has a pole-zero diagram as shown in the following figure and the system is causal, can the inverse system Hi(z), where H(z)Hi(z)=1, be both causal and stable? Clearly justify your answer.