Digital Signal Processing, Fall 2010

Lecture 1: Introduction, Discrete-time signals and systems

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Part I: Introduction

- Introduction (Course overview)
- Discrete-time signals
- Discrete-time systems
- Linear time-invariant systems
- System impulse response
- Linear constant-coefficient difference equations

General information

- Course website
  - http://474043.g.portal.aau.dk/elektronik-it/......
  - http://kom.aau.dk/~zt/cources/digital_signal_processing/
- Textbook:
  - Oppenheim, A.V., Schafer, R.W, "Discrete-Time Signal Processing", 2nd Edition, Prentice-Hall, 1999.
- Readings:
  - Steven W. Smith, "The Scientist and Engineer's Guide to Digital Signal Processing", California Technical Publishing, 1997. http://www.dspguide.com/pdfbook.htm (You can download the entire book!)
  - Kermit Sigmon, "Matlab Primer", Third Edition, Department of Mathematics, University of Florida.
  - V.K. Ingle and J.G. Proakis, "Digital Signal Processing using MATLAB", Bookware Companion Series, 2000.

General information

- Duration
  - 1 ECTS (5 Lectures)
- Prerequisites:
  - Background in advanced calculus including complex variables, Laplace- and Fourier transforms.
- Course type:
  - Study programme course (SE-course), running evaluation
- Lecturer:

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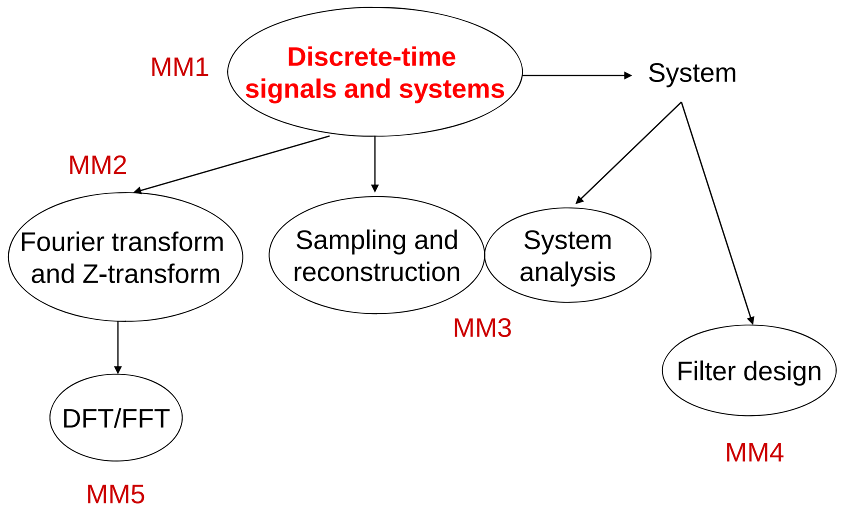
Course at a glance



Course objectives

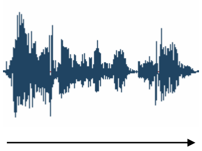
- To understand the concepts of discrete-time signals and systems
- To understand the Z- and the Fourier transform and their inverse
- To understand the relation between digital filters, difference equations and system functions
- To understand the principles of sampling and reconstruction
- To be able to apply digital filters according to known filter specifications
- To know the principles behind the discrete Fourier transform (DFT) and its fast computation
- To be able to apply MATLAB to DSP problems

What is a signal ?

- A flow of information.
- (mathematically represented as) a function of independent variables such as time (e.g. speech signal), position (e.g. image), etc.
- A common convention is to refer to the independent variable as time, although may in fact not.

Example signals

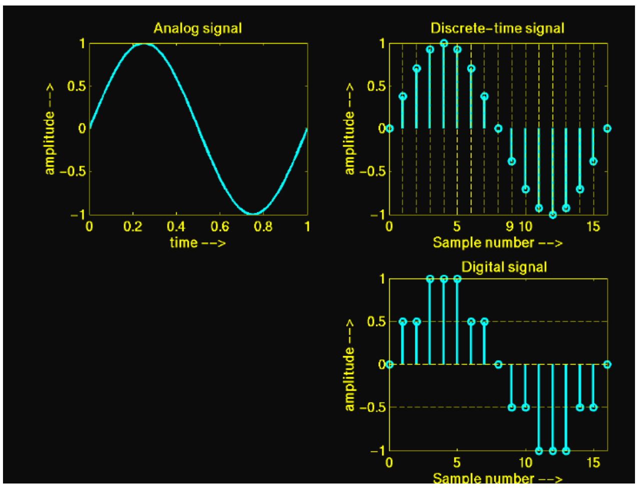
- Speech: 1-Dimension signal as a function of time $s(t)$;.
- Grey-scale image: 2-Dimension signal as a function of space $i(x,y)$
- Video: 3 x 3-Dimension signal as a function of space and time $\{r(x,y,t), g(x,y,t), b(x,y,t)\}$.



Types of signals

- The independent variable may be either continuous or discrete
  - Continuous-time signals
  - Discrete-time signals are defined at discrete times and represented as sequences of numbers
- The signal amplitude may be either continuous or discrete
  - Analog signals: both time and amplitude are continuous
  - Digital signals: both are discrete
- Computers and other digital devices are restricted to discrete time
- Signal processing systems classification follows the same lines

Types of signals



From http://www.ece.rochester.edu/courses/ECE446

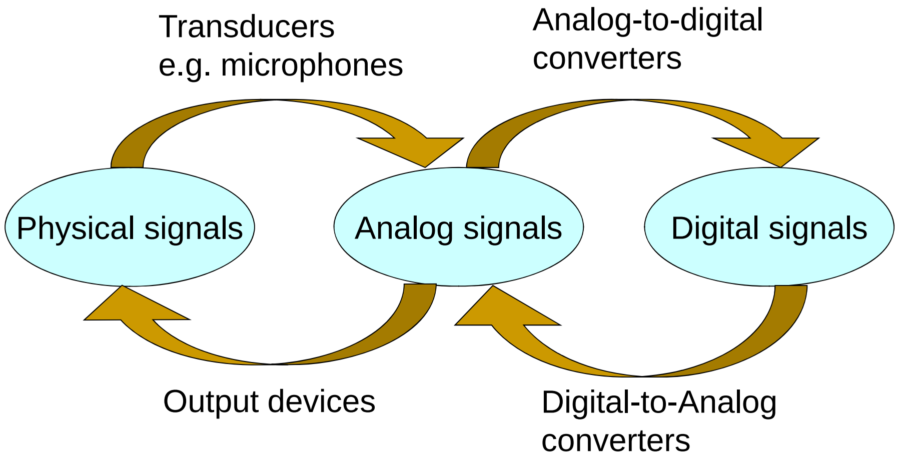
Digital signal processing

- Modifying and analyzing information with computers – so being measured as sequences of numbers.
- Representation, transformation and manipulation of signals and information they contain

Typical DSP system components

- Input lowpass filter to avoid aliasing
- Analog to digital converter (ADC)
- Computer or DSP processor
- Digital to analog converter (DAC)
- Output lowpass filter to avoid imaging

ADC and DAC



Pros and cons of DSP

- Pros
  - Easy to duplicate
  - Stable and robust: not varying with temperature, storage without deterioration
  - Flexibility and upgrade: use a general computer or microprocessor
- Cons
  - Limitations of ADC and DAC
  - High power consumption and complexity of a DSP implementation: unsuitable for simple, low-power applications
  - Limited to signals with relatively low bandwidths

Applications of DSP

- Speech processing
  - Enhancement – noise filtering
  - Coding, synthesis and recognition
- Image processing
  - Enhancement, coding, pattern recognition (e.g. OCR)
- Multimedia processing
  - Media transmission, digital TV, video conferencing
- Communications
- Biomedical engineering
- Navigation, radar, GPS
- Control, robotics, machine vision

History of DSP

- Prior to 1950's: analog signal processing using electronic circuits or mechanical devices
- 1950's: computer simulation before analog implementation, thus cheap to try out
- 1965: Fast Fourier Transforms (FFTs) by Cooley and Tukey – make real time DSP possible
- 1980's: IC technology boosting DSP

Part II: Discrete-time signals

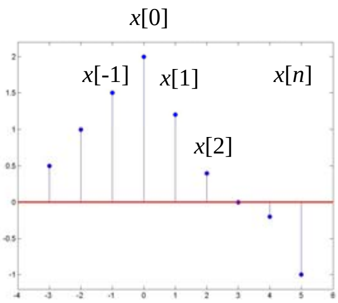
- Introduction
- Discrete-time signals
- Discrete-time systems
- Linear time-invariant systems
- System impulse response
- Linear constant-coefficient difference equations

Discrete-time signals

- Sequences of numbers

$$x = \{x[n]\}, \qquad -\infty < n < \infty$$

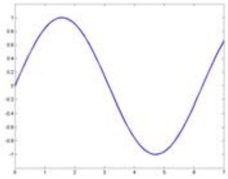
where n is an integer

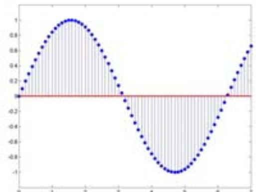


- Periodic sampling of an analog signal

$$x[n] = x_a(nT), \qquad -\infty < n < \infty$$

where T is called the sampling period.





Sequence operations

- The product and sum of two sequences x[n] and y[n]: sample-by-sample production and sum, respectively.
- Multiplication of a sequence x[n] by a number α : multiplication of each sample value by α.
- Delay or shift of a sequence x[n]

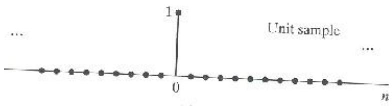
$$y[n] = x[n - n_0]$$

where n is an integer

Basic sequences

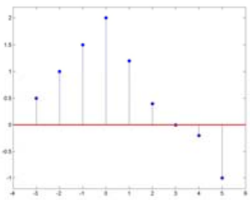
- Unit sample sequence (discrete-time impulse, impulse)

$$\delta[n] = \begin{cases} 0, & n \neq 0, \\ 1, & n = 0, \end{cases}$$



- Any sequence can be represented as a sum of scaled, delayed impulses

$$x[n] = a_{-3}\delta[n+3] + a_{-2}\delta[n+2] + ... + a_5\delta[n-5]$$



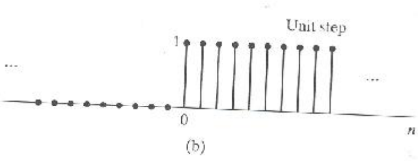
- More generally

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Unit step sequence

- Defined as

$$u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0, \end{cases}$$



- Related to the impulse by

$$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \ldots$$

or

$$u[n] = \sum_{k=-\infty}^{\infty} u[k]\delta[n-k] = \sum_{k=0}^{\infty} \delta[n-k]$$

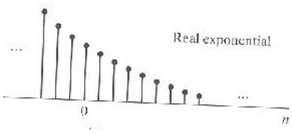
- Conversely,

$$\delta[n] = u[n] - u[n-1]$$

Exponential sequences

- Extremely important in representing and analyzing LTI systems.
- Defined as

$$x[n] = A\alpha^n$$



- If A and α are real numbers, the sequence is real.
- If $0 < \alpha < 1$ and A is positive, the sequence values are positive and decrease with increasing n.
- If $-1 < \alpha < 0$, the sequence values alternate in sign, but again decrease in magnitude with increasing n.
- If $|\alpha| > 1$, the sequence values increase with increasing n.

$x[n] = 2 \cdot (0.5)^n$

$x[n] = 2 \cdot (-0.5)^n$

$x[n] = 2 \cdot 2^n$

Combining basic sequences

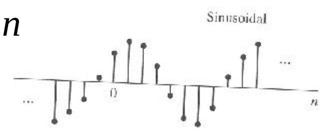
- An exponential sequence that is zero for n<0

$$x[n] = \begin{cases} A\alpha^n, & n \geq 0, \\ 0, & n < 0 \end{cases}$$

$$x[n] = A\alpha^n u[n]$$

Sinusoidal sequences

$$x[n] = A\cos(\omega_0 n + \phi), \qquad \text{for all } n$$



with A and ϕ real constants.

- The $A\alpha^n$ with complex α has real and imaginary parts that are exponentially weighted sinusoids.

If $\alpha = |\alpha| e^{j\omega_0}$ and $A = |A| e^{j\phi}$, then

$$x[n] = A\alpha^n = |A| e^{j\phi} |\alpha|^n e^{j\omega_0 n}$$

$$= |A||\alpha|^n e^{j(\omega_0 n + \phi)}$$

$$= |A||\alpha|^n \cos(\omega_0 n + \phi) + j|A||\alpha|^n \sin(\omega_0 n + \phi)$$

Complex exponential sequence

When $|\alpha|=1$,

$$x[n]=|A|e^{j(\omega_0 n+\phi)}=|A|\cos(\omega_0 n+\phi)+j|A|\sin(\omega_0 n+\phi)$$

- By analogy with the continuous-time case, the quantity ω_0 is called the frequency of the complex sinusoid or complex exponential and ϕ is call the phase.
- *n* is always an integer → differences between discrete-time and continuous-time

An important difference – frequency

- Consider a frequency $(\omega_0+2\pi)$

$$x[n]=Ae^{j(\omega_0+2\pi)n}=Ae^{j\omega_0 n}e^{j2\pi n}=Ae^{j\omega_0 n}$$

- More generally $(\omega_0+2\pi r)$, *r* being an integer,

$$x[n]=Ae^{j(\omega_0+2\pi r)n}=Ae^{j\omega_0 n}e^{j2\pi rn}=Ae^{j\omega_0 n}$$

- Same for sinusoidal sequences

$$x[n]=A\cos[(\omega_0+2\pi r)n+\phi]=A\cos(\omega_0 n+\phi)$$

- So, only consider frequencies in an interval of 2π such as

$$-\pi<\omega_0\le\pi \text{ or } 0\le\omega_0<2\pi$$

An important difference – frequency

- For a continuous-time sinusoidal signal

  $x(t) = A\cos(\Omega_0 t + \phi),$

  as Ω_0 increases, $x(t)$ oscillates more and more rapidly

- For the discrete-time sinusoidal signal

$x[n] = A\cos(\omega_0 n + \phi),$

as ω_0 increases from 0 towards π, $x[n]$ oscillates more and more rapidly

as ω_0 increases from π towards 2π, the oscillations become slower.

Another important difference – periodicity

- In the continuous-time case, a sinusoidal signal and a complex exponential signal are both periodic.
- In the discrete-time case, a periodic sequence is defined as

$$x[n] = x[n+N], \qquad \text{for all } n$$

where the period *N* is necessarily an integer.

- For sinusoid,

  $A\cos(\omega_0 n + \phi) = A\cos(\omega_0 n + \omega_0 N + \phi)$

  which requires that $\omega_0 N = 2\pi k$ or $N = 2\pi k / \omega_0$

  where k is an integer.

Another important difference – periodicity

- Same for complex exponential sequence

  $$e^{j\omega_0(n+N)} = e^{j\omega_0 n},$$

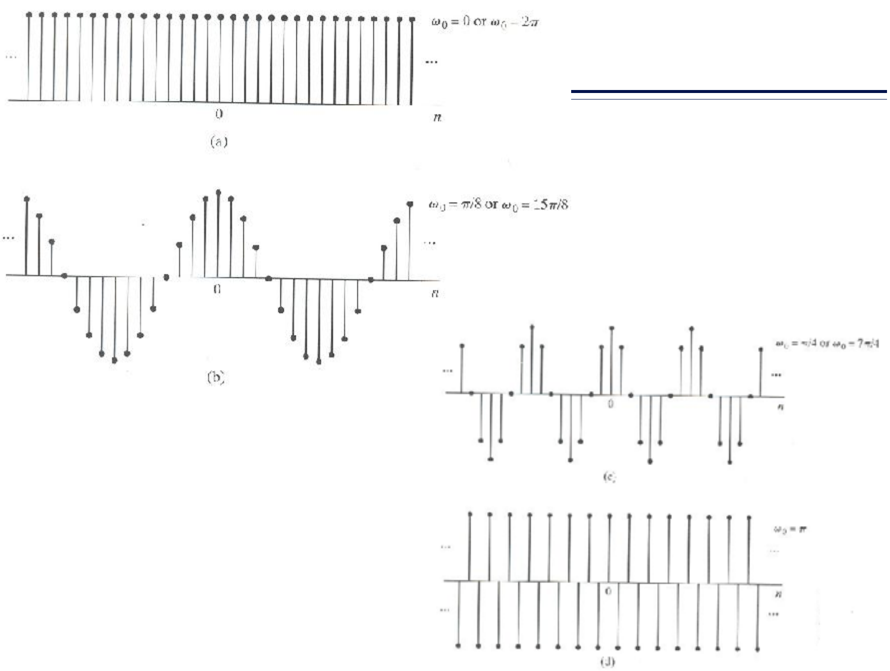
  which is true only for $\omega_0 N = 2\pi k$

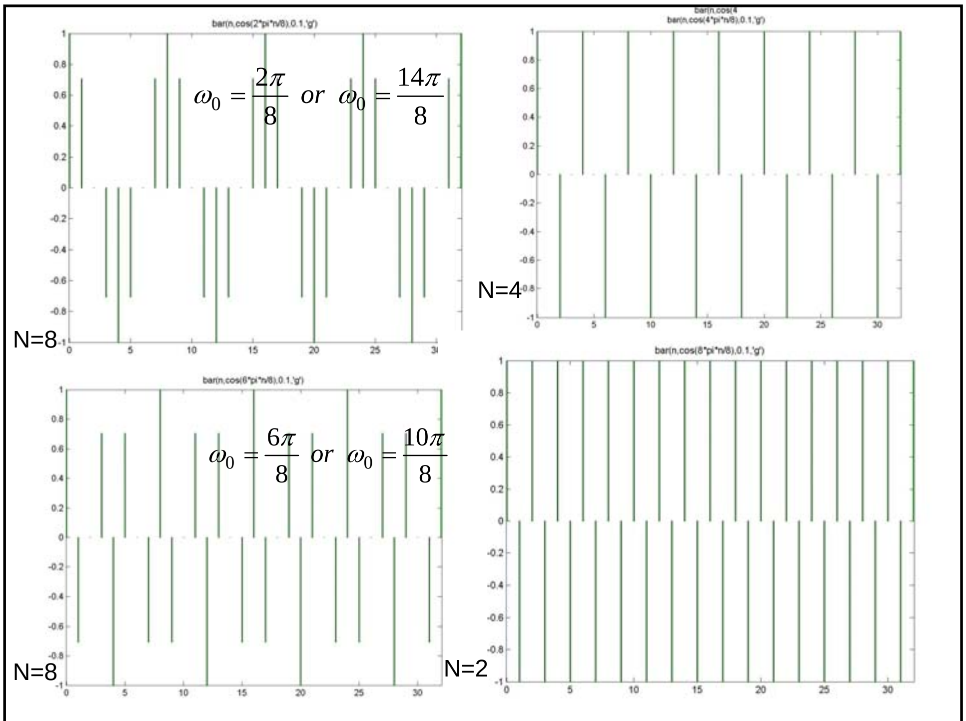
- So, complex exponential and sinusoidal sequences
  - are not necessarily periodic in *n* with period $(2\pi / \omega_0)$
  - and, depending on the value of ω_0 , may not be periodic at all.
- Consider

  $x_1[n] = \cos(\pi n / 4),$ with a period of $N = 8$

  $x_2[n] = \cos(3\pi n / 8),$ with a period of $N = 16$

Increasing frequency → increasing period!





Part III: Discrete-time systems

- Introduction
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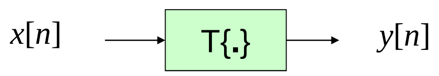
Discrete-time systems

- A transformation or operator that maps input into output

$$y[n] = T\{x[n]\}$$



- Examples:
  - The ideal delay system

$$y[n] = x[n - n_d], \qquad -\infty < n < \infty$$

  - A memoryless system

$$y[n] = (x[n])^2, \qquad -\infty < n < \infty$$

Linear systems

- A system is linear if and only if

additivity property

$$T\{x_1[n] + x_2[n]\} = T\{x_1[n]\} + T\{x_2[n]\} = y_1[n] + y_2[n]$$

and

$$T\{ax[n]\} = aT\{x[n]\} = ay[n]$$ scaling property

where a is an arbitrary constant

- Combined into superposition

$$T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + aT\{x_2[n]\} = ay_1[n] + ay_2[n]$$

Examples

- Accumulator system – a linear system

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

$$y_1[n] = \sum_{k=-\infty}^{n} x_1[k], \qquad y_2[n] = \sum_{k=-\infty}^{n} x_2[k]$$

$$y_3[n] = \sum_{k=-\infty}^{n} (ax_1[k] + bx_2[k]) = ay_1[n] + by_2[n]$$

- A nonlinear system

$$y[n] = \log_{10}(|x[n]|)$$

Consider $x_1[n] = 1$ and $x_2[n] = 10$

Time-invariant systems

- For which a time shift or delay of the input sequence causes a corresponding shift in the output sequence.

$$x_1[n] = x[n - n_0] \Rightarrow y_1[n] = y[n - n_0]$$

- Accumulator system

$$y[n - n_0] = \sum_{k=-\infty}^{n-n_0} x[k]$$

$$y_1[n] = \sum_{k=-\infty}^{n} x_1[k] = \sum_{k=-\infty}^{n} x[k - n_0]$$

$$= \sum_{k_1=-\infty}^{n-n_0} x[k_1] = y[n - n_0]$$

Causality

- The output sequence value at the index $n=n_0$ depends only on the input sequence values for $n<=n_0$.

- Example $y[n] = x[n - n_d], \qquad -\infty < n < \infty$
  - Causal for $n_d>=0$
  - Noncausal for $n_d<0$

Stability

- A system is stable in the BIBO sense if and only if every bounded input sequence produces a bounded output sequence.

- Example

$$y[n] = (x[n])^2, \qquad -\infty < n < \infty$$

stable

Part IV: Linear time-invariant systems

- Course overview
- Discrete-time signals
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Linear time-invariant systems

- Important due to convenient representations and significant applications
- A linear system is completely characterised by its impulse response

$$y[n] = T\{x[n]\} = T\{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\}$$

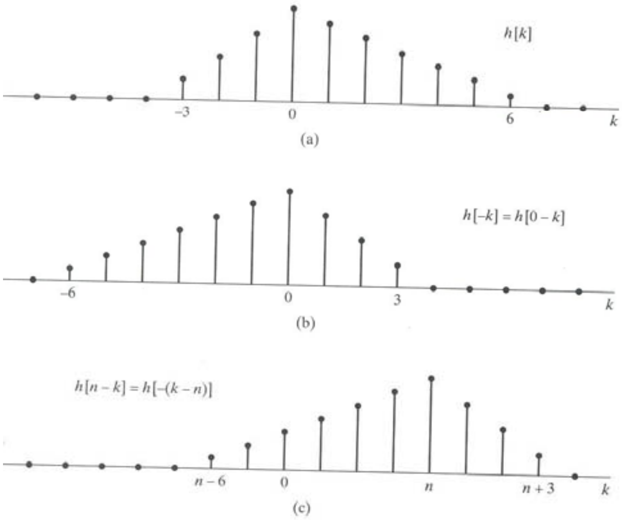
$$= \sum_{k=-\infty}^{\infty} x[k]T\{\delta[n-k]\} = \sum_{k=-\infty}^{\infty} x[k]h_k[n]$$

- Time invariance $h_k[n] = h[n-k]$
- LTI

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$= x[n]*h[n]$$ Convolution sum

Forming the sequence *h*[*n*-*k*]



Computation of the convolution sum

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- Obtain the sequence h[n-k]
  - Reflecting h[k] about the origin to get h[-k]
  - Shifting the origin of the reflected sequence to k=n
- Multiply x[k] and h[n-k] for $-\infty < k < \infty$
- Sum the products to compute the output sample y[n]

Computing a discrete convolution

Example 2.13 pp.26

Impulse response

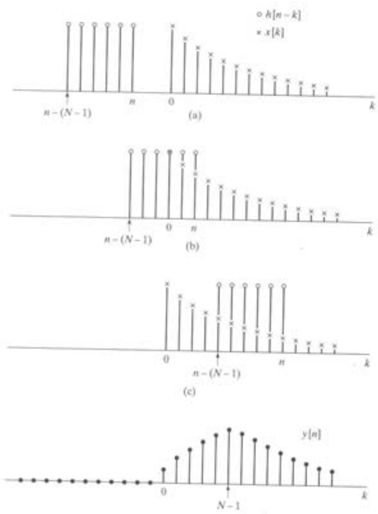
$$h[n] = u[n] - u[n-N]$$

$$= \begin{cases} 1, & 0 \le n \le N-1, \\ 0, & \text{otherwise.} \end{cases}$$

input

$$x[n] = a^n u[n]$$

$$y[n] = \begin{cases} 0, & n < 0, \\ \dfrac{1-a^{n+1}}{1-a}, & 0 \le n \le N-1, \\ a^{n-N+1}\left(\dfrac{1-a^N}{1-a}\right), & N-1 < n. \end{cases}$$



Properties of LTI systems

- Defined by discrete-time convolution
  - Commutative

$$x[n] * h[n] = h[n] * x[n]$$

  - Linear

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

  - Cascade connection (Fig. 2.11 pp.29)

$$h[n] = h_1[n] * h_2[n]$$

  - Parallel connection (Fig. 2.12 pp.30)

$$h[n] = h_1[n] + h_2[n]$$

Properties of LTI systems

- Defined by the impulse response
  - Stable

$$S = \sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

  - Causality

$$h[n] = 0, \qquad n < 0$$

Part V: System impulse response

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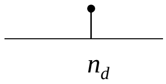
FIR systems – reflected in the $h[n]$

- Ideal delay

$$y[n] = x[n - n_d], \quad -\infty < n < \infty$$

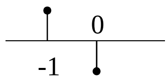
$$\Rightarrow h[n] = \delta[n - n_d], \quad n_d \text{ a positive integer.}$$



- Forward difference

$$y[n] = x[n+1] - x[n]$$

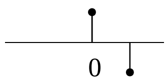
$$\Rightarrow h[n] = \delta[n+1] - \delta[n]$$



- Backward difference

$$y[n] = x[n] - x[n-1]$$

$$\Rightarrow h[n] = \delta[n] - \delta[n-1]$$



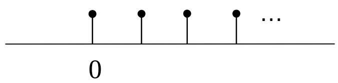
- Finite-duration impulse response (FIR) system
  - The impulse response has only a finite number of nonzero samples.

IIR systems – reflected in the $h[n]$

- Accumulator

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

$$\Rightarrow h[n] = \sum_{k=-\infty}^{n} \delta[k] = u[n]$$



- Infinite-duration impulse response (IIR) system
  - The impulse response is infinitive in duration.
- Stability $S = \sum_{n=-\infty}^{\infty} |h[n]| \overset{?}{<} \infty$
  - FIR systems always are stable, if each of h[n] values is finite in magnitude.
  - IIR systems can be stable, e.g. $h[n] = a^n u[n] \text{ with } |a| < 1$

$$S = \sum_0^{\infty} |a|^n = 1/(1-|a|) < \infty$$

Cascading systems

- Causality $h[n] \overset{?}{=} 0, \quad n < 0$
  - Ideal delay $h[n] = \delta[n - n_d]$

$x[n] \rightarrow$ Forward difference $h[n] = \delta[n+1] - \delta[n]$ $\rightarrow$ One-sameple delay $h[n] = \delta[n-1]$ $\rightarrow y[n]$

$\Downarrow$

$x[n] \rightarrow$ Backward difference $h[n] = \delta[n] - \delta[n-1]$ $\rightarrow y[n]$

  - Any noncausal FIR system can be made causal by cascading it with a sufficiently long delay!

Cascading systems

- Accumulator + Backward difference system

$x[n] \rightarrow$ Accumulator system $h[n] = u[n]$ $\rightarrow$ Backward difference $h[n] = \delta[n] - \delta[n-1]$ $\rightarrow x[n]$

$\Downarrow$

$x[n] \rightarrow x[n]$, $h[n] = \delta[n]$

Inverse system:

$$h[n] * h_i[n] = h_i[n] * h[n] = \delta[n]$$

Part VI: LCCD equations

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LCCD equations

- An important class of LTI systems: input and output satisfy an Nth-order LCCD equations

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$$

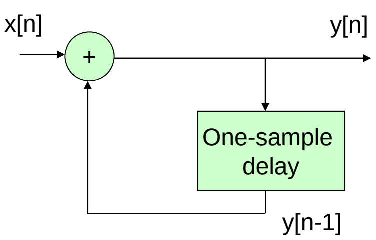
- Difference equation representation of the accumulator

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

$$y[n-1] = \sum_{k=-\infty}^{n-1} x[k]$$

$$y[n] = x[n] + \sum_{k=-\infty}^{n-1} x[k] = x[n] + y[n-1]$$

$$y[n] - y[n-1] = x[n]$$



Recursive representation

MATLAB

- An interactive, matrix-based system for numeric computation and visualization
- Kermit Sigmon, "Matlab Primer", Third Edition, Department of Mathematics, University of Florida.
- Matlab Help (>> *doc*)

Summary

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Course at a glance