Answers, BDSP, MM7

(partial fractions)

1. $$H_c(s) = \frac{s+a}{(s+a)^2+b^2} = \frac{0.5}{s+a-jb} + \frac{0.5}{s+a+jb}$$

Since $\frac{1}{s+\alpha} \Longleftrightarrow e^{-\alpha t}\cdot u(t)$ (Laplace)

$$h_c(t) = \frac{1}{2}\left(e^{-(a+jb)t} + e^{-(a-jb)t}\right)u(t)$$

If $h_1(n) = h_c(nT)$ then

$$h_1(n) = \frac{1}{2}\left[e^{-(a+jb)nT} + e^{-(a-jb)nT}\right]u(n)$$

$$H_1(z) = \frac{0.5}{1-e^{-(a+jb)T}z^{-1}} + \frac{0.5}{1-e^{-(a-jb)T}z^{-1}}$$

$$= \frac{1-e^{-aT}\cos(bT)z^{-1}}{1-2e^{-aT}\cos(bT)z^{-1}+e^{-2aT}z^{-2}}$$

2. $$H(z) = \frac{2}{1-e^{-0.2}z^{-1}} - \frac{1}{1-e^{-0.4}z^{-1}}$$

as $\frac{1}{s+a} \Longleftrightarrow \frac{T_d}{1-e^{-aT_d}z^{-1}}$

we have

$$H_c(s) = \frac{1}{s+0.1} - \frac{0.5}{s+0.2}$$

3. $$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c\left(j\left(\frac{\omega}{T_d} + \frac{2\pi k}{T_d}\right)\right)$$ (2)

If we want $H(e^{j\omega})\Big|_{\omega=0} = H_c(j\Omega_c)\Big|_{\Omega_c=0}$

then it is needed that

$$\sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} H_c\left(j\frac{2\pi k}{T_d}\right) = 0$$

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4. $$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \frac{\pi}{4} \\ 0, & \text{else} \end{cases}$$

$$h_1[n] = h[2n]$$

$$H_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[2n] e^{j\omega n}$$

$$= \sum_{n \text{ even}} h[n] e^{j\frac{\omega n}{2}}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2}\left[h[n] + (-1)^n h[n]\right] e^{j\frac{\omega n}{2}}$$

$$= \frac{1}{2} H(e^{j\frac{\omega}{2}}) + \frac{1}{2} H(-e^{j\frac{\omega}{2}})$$

5. $$H_c(s) = \frac{\Omega_c^2}{s^2 + s\,\Omega_c\sqrt{2} + \Omega_c^2}$$

$\Omega_c = 2\pi 800$ rad/s, 3 dB cutoff freq.

$\omega_c = \frac{\pi}{3}$, using bilinear transf.

a) $\omega_c = T_d \cdot \Omega_c \Rightarrow f_s = \frac{1}{T_d} = \underline{4800 \text{ Hz}}$

b) First $H_c(s)$ is prewarped:

$$\Omega_c' = \frac{2}{T_d} \cdot \tan\left(\frac{\omega_c}{2}\right) = 5543 \text{ rad/s}$$

Then $s = \frac{2}{T_d}\frac{z-1}{z+1}$ is inserted into $H_c(s)$ using Ω_c' instead of Ω_c:

$$H(z) = \frac{\Omega_c'^2}{\left(\frac{2}{T_d}\frac{z-1}{z+1}\right)^2 + \sqrt{2}\,\Omega_c'\left(\frac{2}{T_d}\frac{z-1}{z+1}\right) + \Omega_c'^2}$$

$$= \frac{0.155051 \cdot (z^{-2} + 2z^{-1} + 1)}{1 - 0.6202 z^{-1} + 0.2404 z^{-2}}$$