Digital Signal Processing

http://kom.aau.dk/~zt/cources/DSP/

Exercises of Lecture 7 (MM7)

1. A continuous time system, with impulse response $h_c(t)$ is given by the system function

$$H_c(s) = \frac{s+a}{(s+a)^2 + b^2}$$

Determine $H_1(z)$ for a discrete-time system such that $h_1(n) = h_c(nT)$ (using the impulse invariance method)

2. The system function of a discrete system is $H(z) = \frac{2}{1-e^{-0.2}z^{-1}} - \frac{1}{1-e^{-0.4}z^{-1}}$.

Assume that this system was designed by the impulse invariance method with $T_d = 2$, i.e. $h(n) = 2h_c(2n)$. Determine $H_c(s) = \frac{1}{s+0.1} - \frac{0.5}{s+0.2}$

3. A discrete-time filter with system function H(z) is designed by transforming a continuous-time filter with system function $H_c(s)$. It is desired that

$$H(e^{j\omega})\Big|_{\omega=0} = H_c(j\Omega)\Big|_{\Omega=0}$$

What is the requirement to $H_c(s)$ in order to obtain this (using the impulse invariance method) ?

4. Suppose we are given an ideal lowpass discrete-time filter with the frequency response:

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \pi/4 \\ 0, & \pi/4 < |\omega| \le \pi \end{cases}$$

What is the frequency response $H_1(e^{j\omega})$ for a system having the impulse response $h_1[n] = h[2n]$?

5. An analog 2. order Butterworth lowpass filter is given by the transfer function:

$$H_c(s) = \frac{\Omega_c^2}{s^2 + s\Omega_c\sqrt{2} + \Omega_c^2}$$

$$\Omega_c = 2\pi 800 rad/s$$

Ω_c is the 3 dB cutoff frequency of the analog filter.

A digital filter, $H(z)$, is designed to approximate the analog filter, using the bilinear transformation and having the 3 dB cutoff frequency $\omega_c = \frac{\pi}{3}$

a) Determine the sample frequency given the information about Ω_c and ω_c

b) Use the bilinear transformation to design $H(z)$

Thanks Borge Lindberg for providing part of the exercises and solutions.