

# Solutions, MM<sup>5</sup>

$$1) |H_{ap}(e^{j\omega})|^2 = \left. H(z) \cdot H^*\left(\frac{1}{z^*}\right) \right|_{z=e^{j\omega}}$$

$$H_{ap}(z) \cdot H_{ap}^*\left(\frac{1}{z^*}\right) =$$

$$\frac{(z^{-1} - a^*)(z - a)}{(1 - az^{-1})(1 - a^*z)}$$

$$= \frac{1 - a^*z - az^{-1} - a^*a}{1 - az^{-1} - a^*z - a^*a} = 1$$

$$2) a) H_1(z) = \frac{(1 - 0.5z^{-1})(1 + \frac{1}{4}z^{-2})}{(1 - 0.64z^{-2})}$$

$$H_{ap}(z) = \frac{1 + 4z^{-2}}{1 + \frac{1}{4}z^{-2}}$$

$$b) H_2(z) = \frac{(1 - 0.5z^{-1})}{(1 - 0.64z^{-2})(1 + \frac{1}{4}z^{-2})}$$

$$H_{liq}(z) = (1 + \frac{1}{4}z^{-2})(1 + 4z^{-2})$$

5.3.

$$H(z) = \frac{z^{-1}}{z^{-2} - \frac{10}{3}z^{-1} + 1} = \frac{z^{-1}}{(z^{-1} - 3)(z^{-1} - \frac{1}{3})}$$
$$= \frac{z}{(1-3z)(1-\frac{1}{3}z)}$$

(a) Poles:  $z=3, \frac{1}{3}$       Zeros:  $z=0, \infty$

(b)  $h[n] = -\frac{3}{8} \left(\frac{1}{3}\right)^n u[n] - \left(\frac{3}{8}\right) 3^n u[-n-1]$

5.4.  $H_i(z)$  cannot be causal and stable.

The zero of  $H(z)$  at  $z=\infty$  is a pole of  $H_i(z)$ .

The existence of a pole at  $z=\infty$  implies that the system is not causal.