$\boxed{4.1}$ $x[n] = \cos\left(\frac{n\pi}{4}\right)$, $-\infty < n < \infty$

is obtained by sampling $x_c(t) = \cos(\Omega_0 t)$, $-\infty < t < \infty$

at $F_s = 1000$.

$\omega = \Omega T$, $T = \frac{1}{1000}$

$\omega = \frac{\pi}{4} \Rightarrow \underline{\underline{\Omega_0 = 250\pi}}$

In general:

$\omega = 2\pi k + \frac{\pi}{4}$ gives the same $x[n]$,

k being an integer.

Thus e.g. $\omega = 2\pi + \frac{\pi}{4} \Rightarrow \underline{\underline{\Omega_0 = 2250\pi}}$

$\boxed{4.2}$ $X_c(j\Omega) = 0$, $|\Omega| \geq 2\pi \cdot 10^4$

$x[n] = x_c(nT_s)$

$y[n] = T_s \cdot \sum_{k=-\infty}^{n} x[k] = h[n] * x[n]$

a) $F_s > 2 \cdot F_0 \Rightarrow \underline{\underline{T_s < \frac{1}{2 \cdot 10^{+4}}}}$

b) $y[n] = T_s \cdot \sum_{k=-\infty}^{n} x[k] = T_s \sum_{k=-\infty}^{\infty} x[k]u[n-k]$

$= \cancel{T_s} \sum_{k=-\infty}^{\infty} x[k]h[n-k] \Rightarrow h[n] = \underline{\underline{T_s \cdot u[n]}}$

4.3

$$x[n] \longrightarrow \boxed{\uparrow L} \xrightarrow{x_e[n]} \boxed{h[n]} \longrightarrow y[n]$$

$h[n] = h[-n]$ and $h[n] = 0$, for $|n| > (RL-1)$

a) Since $h[n] = 0$ for $|n| > (RL-1)$ the system becomes causal by delaying $(RL-1)$ samples

$\boxed{4.3}$ Cont.

b) $y[n] = x[n/L]$ for $n = \pm RL$ ~~otherwise~~

$$x_i[n] = \begin{cases} x[n/L], & n = \pm RL \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-kL]$$

$$y[n] = \sum_k x_i[k]\,h[n-k]$$

It is a requirement that

$y[n] = x_i[n]$, when $n = RL$

$$\text{(from (1))} \quad = \sum_m x_i[mL] \cdot h[n-mL]$$

$$\Downarrow$$

$$h[n-mL] = 1, \quad n = mL = RL$$
$$h[RL-mL] = 0, \quad m \neq R$$

$$h[k] = \begin{cases} 1, & k = 0 \\ 0, & k = RL,\ R \neq 0 \end{cases}$$

$\boxed{4.4}$ $x(t) = 10\cos\left(20\pi t - \frac{\pi}{4}\right) - 5\cos(50\pi t)$

a) $F_0 = 25\,Hz$

$\Downarrow$ $\underline{\underline{F_s > 50\,Hz}}$ avoids aliasing

b) We want the 25 Hz to alias to zero.
This is done if $\underline{\underline{F_s = 25\,Hz}}$ $(> 2 \cdot 10)$

c) $x[n] = 10\cos\left(\frac{20\pi n}{25} - \frac{\pi}{4}\right) - 5\cos\left(\frac{50\pi n}{25}\right)$

$$= 10\cos\left(\frac{20\pi n}{25} - \frac{\pi}{4}\right) - 5$$

The reconstructed signal, $y(t)$:

$$y(t) = -5 + 10\cos\left(20\pi t - \frac{\pi}{4}\right)$$

$\Downarrow$ $\underline{\underline{A = -5}}$