

$$\boxed{4.1} \quad x[n] = \cos\left(\frac{n\pi}{4}\right), \quad -\infty < n < \infty$$

is obtained by sampling $x_c(t) = \cos(\Omega_0 t)$, $-\infty < t < \infty$
at $F_s = 1000$.

$$\omega = \Omega T, \quad T = \frac{1}{1000}$$

$$\omega = \frac{\pi}{4} \Rightarrow \underline{\underline{\Omega_0 = 250\pi}}$$

In general:

$\omega = 2\pi k + \frac{\pi}{4}$ gives the same $x[n]$,
 k being an integer.

Thus e.g. $\omega = 2\pi + \frac{\pi}{4} \Rightarrow \underline{\underline{\Omega_0 = 2250\pi}}$

$$\boxed{4.2} \quad X_c(j\Omega) = 0, \quad |\Omega| \geq 2\pi \cdot 10^4$$

$$x[n] = X_c(nT_s)$$

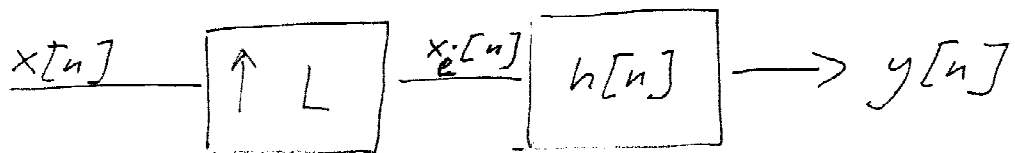
$$y[n] = T_s \sum_{k=-\infty}^{\infty} x[k] = h[n] * x[n]$$

$$a) \quad F_s > 2 \cdot F_0 \Rightarrow \underline{\underline{T_s < \frac{1}{2 \cdot 10^4}}}$$

$$b) \quad y[n] = T_s \sum_{k=-\infty}^{\infty} x[k] = T_s \sum_{k=-\infty}^{\infty} x[k] u[n-k]$$

$$= \sum_{k=-\infty}^{\infty} x[k] h[n-k] \Rightarrow \underline{\underline{h[n] = T_s \cdot u[n]}}$$

4.3



$$h[n] = h[-n] \text{ and } h[n] = 0, \text{ for } |n| > (RL-1)$$

a) Since $h[n] = 0$ for $|n| > (RL-1)$ the system becomes causal by delaying $(RL-1)$ samples

4.3 Const.

b) $y[n] = x[n/L]$ for $n = \pm RL$ ~~otherwise~~

$$x_i[n] = \begin{cases} x[n/L], & n = \pm RL \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$= \sum_{k=-\infty}^{\infty} x[k] \delta[n - kL]$$

$$y[n] = \sum_k x_i[k] h[n - k]$$

It is a requirement that

$$y[n] = x_i[n], \text{ when } n = RL$$

(from ①)
$$= \sum_m x_i[mL] \cdot h[n - mL]$$

$$\Downarrow \begin{cases} h[n - mL] = 1, & n = mL = RL \\ h[RL - mL] = 0, & m \neq R \end{cases}$$

$$h[k] = \begin{cases} 1, & k = 0 \\ 0, & k = RL, R \neq 0 \end{cases}$$

$$4.4 \quad x(t) = 10 \cos\left(20\pi t - \frac{\pi}{4}\right) - 5 \cos(50\pi t)$$

$$a) \quad F_0 = 25 \text{ Hz}$$

\Downarrow
 $F_s > 50 \text{ Hz}$ avoids aliasing

b) We want the 25 Hz to alias to zero.
This is done if $F_s = 25 \text{ Hz}$ ($> 2 \cdot 10$)

$$c) \quad X[n] = 10 \cos\left(\frac{20\pi n}{25} - \frac{\pi}{4}\right) - 5 \cos\left(\frac{50\pi n}{25}\right) \\ = 10 \cos\left(\frac{20\pi n}{25} - \frac{\pi}{4}\right) - 5$$

The reconstructed signal, $y(t)$:

$$y(t) = -5 + 10 \cos\left(\frac{20\pi t}{25} - \frac{\pi}{4}\right)$$

\Downarrow
 $A = -5$