

Digital Signal Processing

<http://kom.aau.dk/~zt/courses/DSP/>

Exercises of Lecture 3 (MM3)

Exercise 3.1. Find the Z-transform and ROC of the following sequences

- a) $(\frac{1}{2})^n u[n]$
- b) $-(\frac{1}{2})^n u[-n-1]$
- c) $(\frac{1}{2})^n u[-n]$
- d) $\delta[n-1]$
- e) $\delta[n+1]$
- f) $(\frac{1}{2})^n [u[n] - u[n-10]]$
- g) $x[n] = \alpha^{|n|}$

Exercise 3.2. The input to a LTI-system is $u[n]$ and the output $y[n]$ is $y[n] = (\frac{1}{2})^{n-1} u[n+1]$.

Find the transfer function, $H(z)$.

Exercise 3.3. Find the Z-transform of the sequence

$$x[n] = \begin{cases} n, & 0 \leq n \leq N-1, \\ N, & N \leq n. \end{cases}$$

Hint: Express $x[n]$ using $u[n]$.

Exercise 3.4. Given the Z-transform $X(z) = \frac{1}{(1 + \frac{1}{2}z^{-1})}$, $|z| > \frac{1}{2}$. Use power series expansion

(long division) to find $x[n]$.

Exercise 3.5. Given the Z-transform $X(z) = \frac{1 - \frac{1}{2}z^{-1}}{(1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2})}$, $|z| > \frac{1}{2}$. Try both using partial

fraction expansion and power series expansion (long division) to find $x[n]$.

Exercise 3.6. Given the transfer function $H(z) = \frac{1 + z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})}$ of a causal LTI system.

Is the system stable? (First find the ROC)