BDSP, Jan. 13, 2004 <u>Ordinary</u>

1. $H_c(s) = \frac{2}{s+2}$

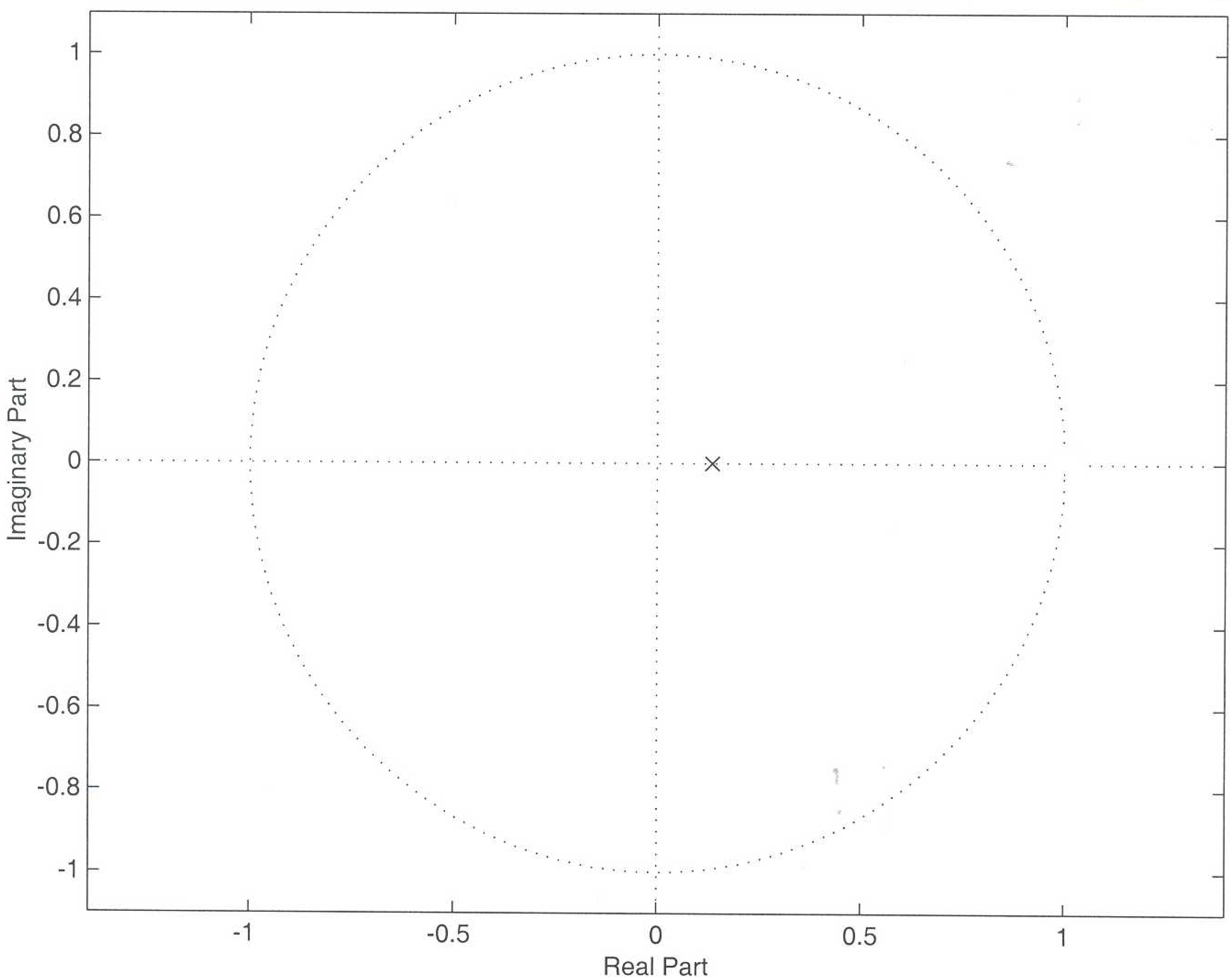
Impulse invariance method implied

$h(n) = T \cdot h_c(nT)$

$= T \cdot 2 \cdot e^{-2Tn} = 2 \cdot e^{-2n}$

1.1 $H(z) = \frac{2}{1 - e^{-2} z^{-1}}$

1.2 Pole at $z = e^{-2} = \underline{\underline{0.1353}}$



2. $H_1(z) = \dfrac{0.5}{1-0.5z^{-1}}$, $H_2(z) = \dfrac{0.5j}{1-0.5jz^{-1}}$

2.1 $H(z) = H_1(z) + H_2(z)$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{2}j}{1-\frac{1}{2}jz^{-1}}$$

$$= \frac{\frac{1}{2}\cdot\left(1-\frac{1}{2}jz^{-1}\right) + \frac{1}{2}j\left(1-\frac{1}{2}z^{-1}\right)}{1-\left(\frac{1}{2}+\frac{1}{2}j\right)z^{-1} + \frac{1}{4}jz^{-2}}$$

$$= \frac{\frac{1}{2}+\frac{1}{2}j - \frac{1}{2}jz^{-1}}{1-\left(\frac{1}{2}+\frac{1}{2}j\right)z^{-1} + \frac{1}{4}jz^{-2}}$$

2.2 $h(n) = \frac{1}{2}\cdot\left(\frac{1}{2}\right)^n u(n) + \frac{1}{2}j\left(\frac{1}{2}j\right)^n u(n)$

$$= \left(\frac{1}{2}\right)^{n+1} u(n) + \left(\frac{1}{2}j\right)^{n+1}\cdot u(n)$$

2.3 $y(n) = \left(\frac{1}{2}+\frac{1}{2}j\right)x(n) - \frac{1}{2}jx(n-1)$
$+ \left(\frac{1}{2}+\frac{1}{2}j\right)y(n-1) - \frac{1}{4}jy(n-2)$

2.4 $x(n) = 2\cdot\delta(n-1) - \delta(n-2)$

$y(0) = 0$

$y(1) = \left(\frac{1}{2}+\frac{1}{2}j\right)\cdot 2 = 1+j$

$y(2) = \left(\frac{1}{2}+\frac{1}{2}j\right)\cdot(-1) - \frac{1}{2}j\cdot 2 + \left(\frac{1}{2}+\frac{1}{2}j\right)(1+j)$

$= -\frac{1}{2}-\frac{1}{2}j$

2.4 (Cont)

$$y[3] = +\frac{1}{4} - \frac{1}{4}j$$

$$y[4] = +\frac{1}{8} + \frac{1}{8}j$$

3. $h(n) = [0.5 \quad 0.5]$, $f_s = 44.1\ kHz$

$x(n) = [1 \quad 2]$

3.1 $$y(n) = h(n) * x(n)$$
$$= [0.5 \quad 1.5 \quad 1]$$

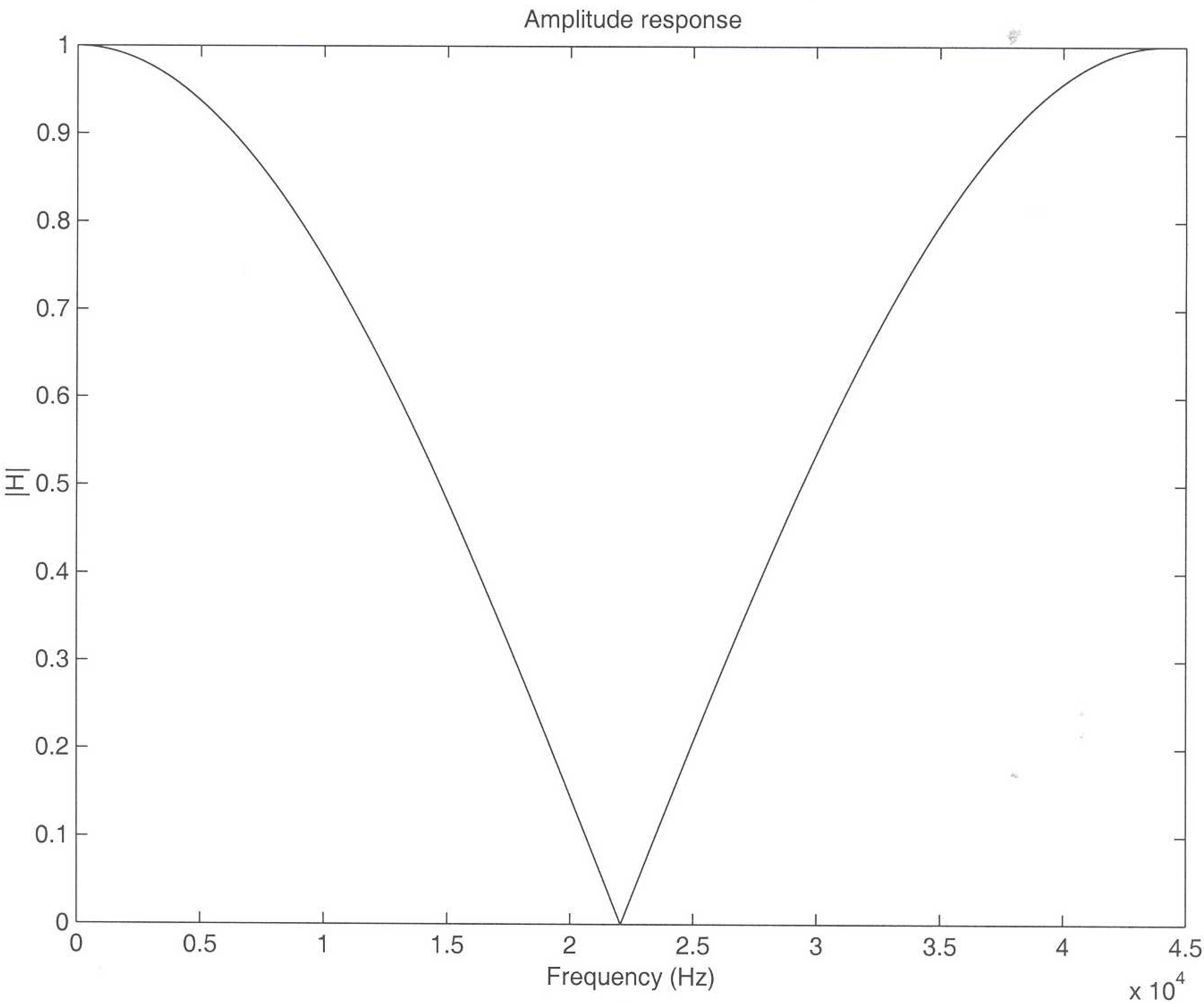
3.2 $$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h(n) \cdot e^{-jnw} = \underline{\underline{\frac{1}{2} + \frac{1}{2}e^{-jw}}}$$

3.3 $$H(e^{jw}) = \sum_{n=0}^{1} h(n) \cdot e^{-jn \cdot \frac{2\pi f}{44100}}$$

$$= \frac{1}{2} + \frac{1}{2}\cos\left(\frac{-2\pi f}{44100}\right) + \frac{1}{2}j\sin\left(\frac{-2\pi f}{44100}\right)$$

(Sketched at next page)

3.4 It is seen that the filter is a low-pass filter



Amplitude response
|H|
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Frequency (Hz)
x 10^4

3.5 $H(k) = \sum_{n=0}^{N-1} h(n)\, e^{-j\left(\frac{2\pi}{N}\right)kn}$

$N = 2$

$H(0) = \frac{1}{2}(\cos(0) + j\sin(0))$
$+ \frac{1}{2}(\cos(0) + j\sin(0)) = 1$

$H(1) = 0$

$X(0) = 3, \quad X(1) = -1$

3.6 $Y(k) = H(k) \cdot X(k) = [3\ 0]$

3.7 $y_1[n] = IDFT\{Y(k)\}$

$= \frac{1}{N} \cdot \sum_{k=0}^{N-1} Y(k) \cdot e^{j\frac{2\pi}{N} \cdot kn}$

$y_1[0] = \frac{1}{2} \cdot 3 \cdot (\cos(0) + j\sin(0)) = 1.5$

$y_1[1] =$ — " — 1.5

3.8 $y_1[n]$ is the result of a circular convolution ⓝ which is different from the linear convolution.
The span of the circular convolution needs to be longer (N):

$N \geq N_1 + N_2 - 1$

$\Downarrow$

$N \geq 3$

4. $H(z) = \dfrac{3 - \frac{10}{6} z^{-1} + \frac{1}{12} z^{-2}}{1 + \frac{1}{12} z^{-1} - \frac{1}{12} z^{-2}}$

4.1 zeros at $\frac{1}{2}, \frac{1}{18}$

poles at $\frac{1}{4}, -\frac{1}{3}$

4.2 Yes, the filter is stable as the poles are inside the UC

4.3

$$H(z) = \frac{3 \cdot (1 - \frac{1}{2} z^{-1})(1 - \frac{1}{18} z^{-1})}{(1 - \frac{1}{4} z^{-1})(1 + \frac{1}{3} z^{-1})}$$

$$= 3 \left(1 - \frac{1}{2} z^{-1}\right) \cdot \left[\frac{A(1 + \frac{1}{3} z^{-1}) + B(1 - \frac{1}{4} z^{-1})}{(1 - \frac{1}{4} z^{-1})(1 + \frac{1}{3} z^{-1})}\right]$$

$$= \text{— '' —} \left[\frac{A + B + z^{-1} \cdot \left(\frac{A}{3} - \frac{B}{4}\right)}{\text{— '' —}}\right]$$

$$A + B = 1 \quad \wedge \quad \frac{A}{3} - \frac{B}{4} = -\frac{1}{18}$$

$\Downarrow$

$$A = \frac{1}{3} \quad \wedge \quad B = \frac{2}{3}$$

$\Downarrow$

$$H(z) = \left(1 - \frac{1}{2} z^{-1}\right) \cdot \left(\frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{2}{1 + \frac{1}{3} z^{-1}}\right)$$

$\Downarrow$

$$h(n) = \left(\frac{1}{4}\right)^n u(n) + 2\left(\frac{-1}{3}\right)^n u(n) - \frac{1}{2}\left(\frac{1}{4}\right)^{n-1} u(n-1) - \left(\frac{-1}{3}\right)^{n-1} u(n-1)$$

4.4 $y(n) = 3 \cdot x(n) - \frac{10}{6} x(n-1) + \frac{1}{12} x(n-2)$

$- \frac{1}{12} y(n-1) + \frac{1}{12} y(n-2)$

4.5. $h(0) = 3$

$h(1) = 0 - \frac{10}{6} - \frac{1}{12} \cdot 3 = -1.9167$

$h(2) = 0.4931$

$h(3) = -0.2008$

4.6 $G(z) = \frac{1}{H(z)}$

$G(z)$ is stable as all the zeros of $H(z)$ are inside the UC. These become the poles of $G(z)$.

5. $h_d(n) = \dfrac{\sin\left(\frac{\pi}{4}(n-2)\right)}{(n-2)\cdot \pi}$

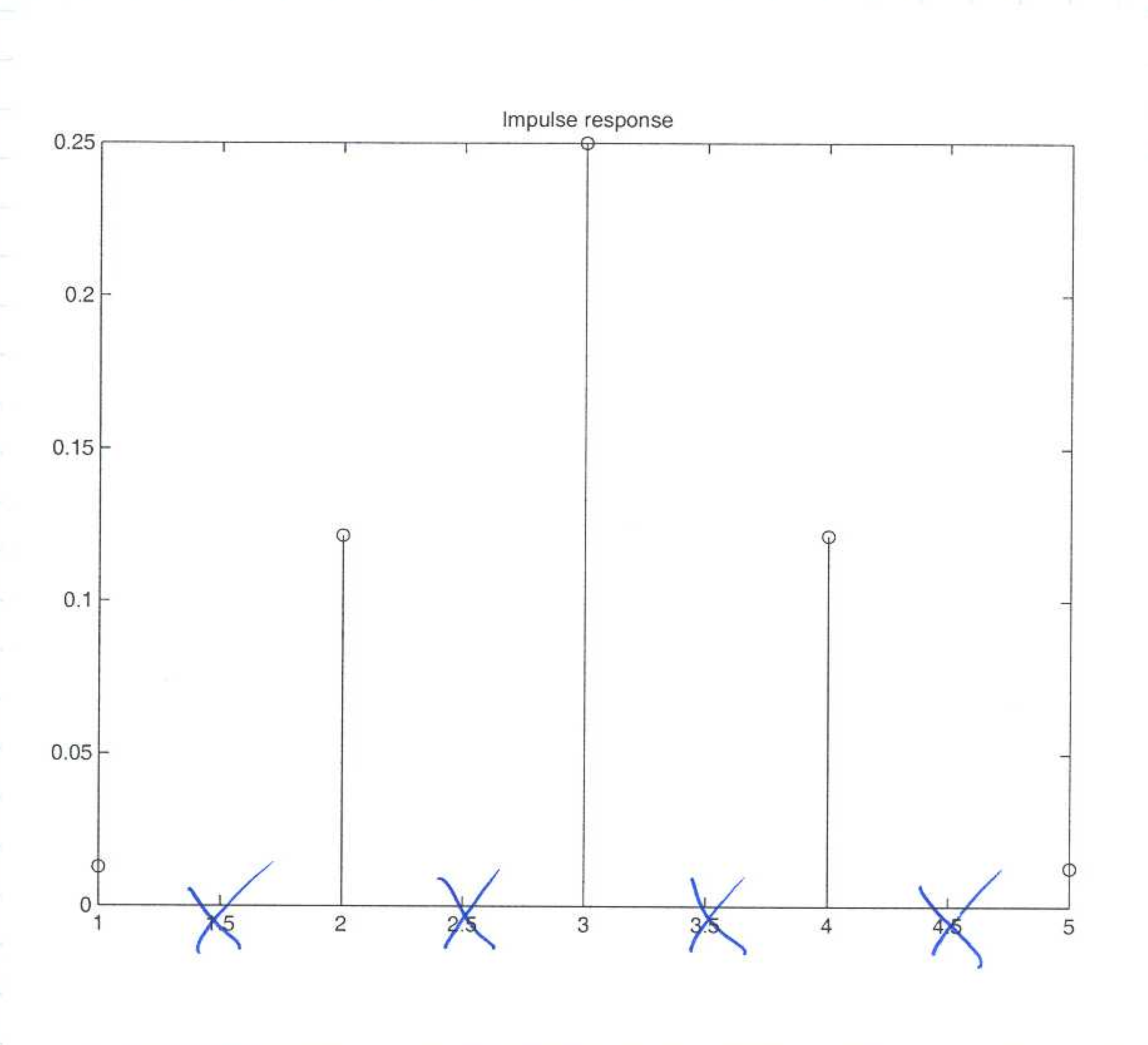
$$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{M}\right), & 0 \le n \le M \\ 0, & \text{else} \end{cases}$$

5.1 $h(n) = h_d(n) \cdot w(n)$

$$h(0) = h(4) = \frac{\sin\left(\frac{\pi}{2}\right)}{2\pi} \cdot 0.08 = 0.0127$$

$$h(1) = h(3) = \frac{\sin\left(\frac{\pi}{4}\right)}{\pi} \cdot 0.54 = 0.1215$$

$$h(2) = \lim_{n \to 2} \left(\frac{\sin\left(\frac{\pi}{4}(n-2)\right)}{(n-2)\pi} \right) \cdot 1 = 0.25$$



5.2 $H(z) = 0.0127 + 0.1215 z^{-1} + 0.25 z^{-2}$

$+ 0.1215 z^{-3} + 0.0127 z^{-4}$

5.3 An FIR-filter is always stable independent of the position of the zeros.