Digital Signal Processing

http://kom.aau.dk/~zt/cources/DSP_E/

Exercises of Lecture 10 (MM10)

INSTITUTE OF ELECTRONIC SYSTEMS
AALBORG UNIVERSITY

Written exam in

Basic Digital Signal Processing (2 Modules)

Ordinary Curriculum

Tuesday, January 13, 2004
9.00 – 12.00

All ordinary tools may be used.
Remember to put your full name on every sheet you deliver.

The problems given contribute to the grade according to the following table:

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
Contribution	7 %	18 %	35 %	25 %	15 %

Problem 1

The transfer function of an analog filter is given by:

$$H_c(s) = \frac{2}{s+2}$$

Question 1.1 Design a digital filter $H(z)$ from $H_c(s)$ using the impulse invariance method. Use a sampling frequency of 1 Hz.

Question 1.2 Illustrate the position of the pole of $H(z)$ in the z-plane

Problem 2

A linear time invariant system is shown in Figure 1.

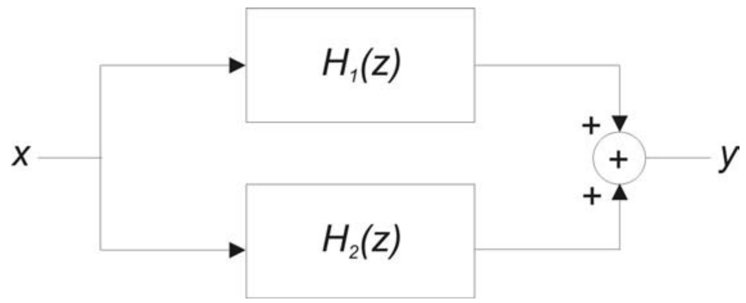

Figure 1 Linear system

The two linear time invariant systems, $H_1(z) = \frac{0.5}{1-0.5z^{-1}}$ and $H_2(z) = \frac{0.5j}{1-0.5jz^{-1}}$, are connected together to form $H(z)$ in the system of Figure 1. $j = \sqrt{-1}$

Question 2.1 Find the transfer function, $H(z)$, of the system

Question 2.2 Find the impulse response, $h[n]$, of the system

Question 2.3 Find the difference equation for the system, expressed by the input x and the output y

Question 2.4 For the input signal $x[n] = 2\delta[n-1] - \delta[n-2]$, find the first five values of the output: $y[0], y[1], y[2], y[3], y[4]$

Problem 3

A digital filter has the impulse response $h[n] = [0.5 \quad 0.5]$ and is implemented at a sampling frequency of $f_s = 44.1kHz$

An input signal $x[n] = [1.0 \quad 2.0]$ is sent through the filter, giving the output $y[n]$.

Question 3.1 Find the output $y[n]$

Question 3.2 Find the Discrete Time Fourier Transform $H(e^{j\omega})$ of $h[n]$

Question 3.3 Find the amplitude response of the filter, and sketch it as a function of frequency f

Question 3.4 Is the filter a low-pass or a high-pass filter ? (Justify your answer)

Question 3.5 Calculate the two 2-point DFT's $X[k]$ and $H[k]$ of $x[n]$ and $h[n]$ respectively.

Question 3.6 Find $Y[k] = H[k] \cdot X[k]$

Question 3.7 Find $y_1[n]$ as the inverse DFT of $Y[k]$

Question 3.8 Explain the difference between $y_1[n]$ and $y[n]$

Problem 4

A digital filter is characterised by the following transfer function:

$$H(z) = \frac{3 - \frac{10}{6}z^{-1} + \frac{1}{12}z^{-2}}{1 + \frac{1}{12}z^{-1} - \frac{1}{12}z^{-2}}$$

Question 4.1 Find the zeros and poles of the digital filter $H(z)$

Question 4.2 Is the filter stable ?

Question 4.3 Find the impulse response, $h[n]$, of the filter

Question 4.4 Find the difference equation for the filter, expressed by the input x and the output y

Question 4.5 Find $h[0]$, $h[1]$, $h[2]$ and $h[3]$

Define $G(z) = \frac{1}{H(z)}$, the inverse transfer function of $H(z)$.

Question 4.6 Is $G(z)$ stable ? (Justify your answer)

Problem 5

A fourth order Type I linear phase FIR filter, *h[n]*, is to be designed using the window method.

The ideal (wanted) impulse response of the filter is defined as (M is the filter order):

$$h_d[n] = \frac{\sin(n - \frac{M}{2})\frac{\pi}{4}}{(n - \frac{M}{2})\pi}$$

Question 5.1 Find and sketch *h[n]* using a Hamming window

Question 5.2 Find the transfer function, *H(z),* of the filter

Question 5.3 Is the filter stable ? (Justify your answer)